Appendix to
"Monetary Policy and Credit Supply Adjustment with Endogenous Default and Prepayment"

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A Model Solution

In this section, I derive first-order necessary conditions (FONCs) for all agents in the model. I add further derivations and explanations for those of interest.

A.1 Impatient Household Optimality

The Lagrangian for the representative impatient household is

$$
\begin{aligned}
\mathcal{L} = \sum_{t=0}^{\infty} \beta^t \{ u(c_t, h_t, n_t) + \lambda_t \big[& (1-\tau_y) w_t n_t - (1-\Delta_t)\pi_t^{-1} x_{m,t-1} + \tau_y (1-\Delta_t)(x_{m,t-1} - \varphi)\pi_t^{-1} \\
& + (1-\Delta_t)\varrho_t \left(m_t^* - (1-\varphi)\pi_t^{-1} m_{t-1} \right) - \delta_h q_t^h h_{t-1} \\
& - \varrho_t q_t^h ((1-\Delta_t) h_t^* - (1-G_{m,t}) h_{t-1}) - \bar{\Delta}_t q_t^h h_t^* \\
& - \left(\Psi(\varrho_t) - \bar{\Psi}_t \right) m_t^* + T_t - c_t \big] \\
& + \lambda_t \lambda_t^{KM} \left(\theta_t^{LTV*} q_t^h h_t^* - m_t^* \right) \\
& + \lambda_t \Omega_t^m \left(-\tilde{\varrho}_t m_t^* - (1-\tilde{\varrho}_t)(1-\varphi)\pi_t^{-1} m_{t-1} + m_t \right) \\
& + \lambda_t \Omega_t^x \left(x_{m,t} - \tilde{\varrho}_t r_t^* m_t^* - (1-\tilde{\varrho}_t)(1-\varphi)\pi_t^{-1} x_{m,t-1} \right) \\
& + \lambda_t \Omega_t^h \left(\tilde{\varrho}_t h_t^* + (1-\tilde{\varrho}_t)(1-G_{m,t}) h_{t-1} - h_t \right) \\
\}
\end{aligned}
$$

where $u(\cdot)$ is the related utility function, and variables are defined as in the main text. Interpretation of the multipliers are:

- λ_t: marginal utility of consumption, equals $\frac{\partial u}{\partial c}$.
- λ_t^{KM}: The marginal value of loosening one unit of collateral constraint.
- Ω_t^m: the marginal value of increasing one unit of mortgage. Can be thought of as the **cost** increasing the mortgage.
- Ω_t^x: the marginal value of increasing one unit of promised payment next period. Can be thought of as the **cost** increasing promised payment.
- Ω_t^h: the marginal value of increasing one unit of housing stock at t. Can be thought as the **price** of housing stock for the borrowers.[1]

The important first order condition for prepayment threshold is[2]

$$
\begin{aligned}
\kappa_t^* m_t^* = & (1-\Omega_t^m)(m^* - (1-\varphi)\pi^{-1} m_{t-1}) \\
& - \Omega_t^x (r_{m,t}^* m_t^* - (1-\varphi)\pi_t^{-1} x_{m,t-1}) \\
& - (q_t^h - \Omega_{b,t}^h)(h_t^* - \tilde{h}_t)
\end{aligned}
$$

[1] From FONC of h^* presented later, it can be found out that $\Omega_t^h < q_t^h$, because of the heterogeneous preference and the collateral constraint.

[2] Here I use the fact that the refinancing rate is a function of the threshold policy variable κ_t^p.

where $\tilde{h}_t = \frac{1-G_{m,t}}{1-\Delta_t} h_{t-1}$. Rearrange term and divide through by m_t^*, it becomes

$$\begin{aligned}\kappa_t^* &= (1-\Omega_t^m)\left(1-\frac{(1-\varphi)\pi_t^{-1}m_{t-1}}{m_t^*}\right)\\ &\quad -\Omega_t^x\left(r_{m,t}^* - \frac{(1-\varphi)\pi_t^{-1}x_{t-1}}{m_t^*}\right)\\ &\quad -(q_t^h-\Omega_t^h)(h_t^*-\tilde{h}_t)/m_t^*.\end{aligned}$$

Using the fact that $x_{m,t-1} = r_{m,t-1}m_{t-1}$, it can be written as

$$\begin{aligned}\kappa_t^* = &\underbrace{(1-\Omega_t^m-\Omega_t^x r_{m,t-1})\left(1-\frac{(1-\varphi)\pi_t^{-1}m_{t-1}}{m_t^*}\right)}_{\text{New debt incentive}}\\ &-\underbrace{\Omega_t^x\left(r_{m,t}^*-r_{m,t-1}\right)}_{\text{Interest incentive}}-\underbrace{(q_t^h-\Omega_{t-1}^h)(h_t^*-\tilde{h}_t)/m_t^*}_{\text{Price incentive}}\end{aligned}$$

which is the result shown in the main text.

The first order condition for the default threshold $\omega_{m,t}^*$ is

$$\begin{aligned}\omega_{m,t}^* = &\left\{\left[\varrho_t q_t^h + (1-\varrho_t)\Omega_t^h\right]h_{t-1}\right\}^{-1}\cdot\\ &\Big(\left(\pi_t^{-1}x_{m,t-1}+(1-\varphi)\pi_t^{-1}m_{t-1}-\tau_y\pi_t^{-1}\left(x_{t-1}-\varphi m_{t-1}\right)\right)\\ &-(1-\Omega_{m,t})\left(\varrho_t m_t^* + (1-\varrho_t)(1-\varphi)\pi_t^{-1}m_{t-1}\right)\\ &+\Omega_t^x\left(\varrho_t r_{m,t}^* m_t^* + (1-\varrho_t)(1-\varphi)\pi_t^{-1}x_{m,t-1}\right)\\ &+\varrho_t h_t^*\left(q_t^h-\Omega_t^h\right)\Big)\end{aligned}$$

Other first order conditions are

$$\begin{aligned}
&(c_t) && \lambda_t = && u_{c,t}\\
&(n_t) && u_{n,t} = && -\lambda_t(1-\tau_y)w_t\\
&(h_t) && \Omega_t^h = && \frac{u_{h,t}}{\lambda_t} + \mathbb{E}_t\Lambda_{t+1}\left[(\varrho_{t+1}(1-G_{m,t+1})-\delta_h)q_{t+1}^h + (1-\varrho_{t+1})(1-G_{m,t+1})\Omega_{t+1}^h\right]\\
&(h_t^*) && 1-\frac{\Omega_t^h}{q_t^h} = && \lambda_t^{KM}\theta_t^{LTV*}\\
&(m_t) && \Omega_t^m = && \mathbb{E}_t\pi_{t+1}^{-1}\left[(1-\Delta_{t+1})\tau_y\varphi + \tilde{\varrho}_{t+1}(1-\varphi) + (1-\tilde{\varrho}_{t+1})(1-\varphi)\Omega_{t+1}^m\right]\\
&(m_t^*) && 1 = && \lambda_t^{KM} + \Omega_t^m + \Omega_t^x r_{m,t}^*\\
&(x_{m,t}) && \Omega_t^x = && \mathbb{E}_t\Lambda_{t+1}\pi_{t+1}^{-1}(1-\Delta_{t+1})\left[(1-\tau_y) + (1-\varrho_{t+1})(1-\varphi)\Omega_{t+1}^x\right]
\end{aligned}$$

A.2 Patient Household Optimality

The patient household's problem is relatively standard. The Lagrangian for the representative patient household is

$$\begin{aligned}\mathcal{L} = \sum_{t=0}^{\infty}\beta^t\Big\{&u'(c_t',\bar{h}_t',n_t') + \lambda_t'\big[(1-\tau_y)w_t'n_t' + (1+r_{t-1}')\pi_t^{-1}d_{t-1}' - d_t - \delta_h q_t^h h_{t-1}'\\ &+(R_{t-1}^{ib}\pi_t^{-1}b_{t-1}'-b_t') - (h_t'-h_{t-1}') + \Pi_t' + T_t' - c_t'\big]\Big\}\end{aligned}$$

Related first order conditions are

$$
\begin{aligned}
&(c_t') && \lambda_t' = && u_{c',t}' \\
&(n_t) && u_{n',t}' = && -\lambda_t'(1-\tau_y)w_t' \\
&(\bar{h}_t') && q_t^h = && \frac{u_{h',t}'}{\lambda_t'} + \mathbb{E}_t\Lambda_{t+1}'(1-\delta_h)\, q_{t+1}^h \\
&(b_t') && 1 = && \mathbb{E}_t\left\{\Lambda_{t,t+1}' R_t^{ib}\pi_{t+1}^{-1}\right\}
\end{aligned}
$$

A.3 Entrepreneurs Optimality

Given the setup in the main text, the individual entrepreneur's optimization problem is essentially identical, thus the index i is suppressed, and $\kappa_t(i) = \kappa_t$ and $\bar{\omega}_{t+1}(i) = \bar{\omega}_{t+1}$. The problem can be written as

$$\max_{\kappa_t, \bar{\omega}_{t+1}} NW_t \mathbb{E}_t\left[\gamma(1-\Gamma_{t+1}(\bar{\omega}_{t+1}))R_{t+1}^k \kappa_t\right]$$

subject to the participation constraint

$$\tilde{R}_{e,t}^{I,a}(\kappa_t - 1) = \mathbb{E}_t\left\{\kappa_t R_{t+1}'^{k}\left(\Gamma_{t+1} - \mu G_{t+1}\right)\right\}.$$

The Lagrangian is

$$\mathcal{L} = NW_t\mathbb{E}_t\left[\gamma(1-\Gamma_{t+1})R_{t+1}^k\kappa_t\right] + \lambda_t^E\left\{\mathbb{E}_t\left[\kappa_t R_{t+1}^k(\Gamma_{t+1}-\mu G_{t+1})\right] - \tilde{R}_{e,t}^{I,a}(\kappa_t - 1)\right\}$$

The FONCs are

$$
\begin{aligned}
&(\kappa_t) && 0 = \gamma NW_t\mathbb{E}_t\left[(1-\Gamma_{t+1})R_{t+1}^k\right] + \lambda_t^E\left\{\underbrace{\mathbb{E}_t\left[R_{t+1}^k(\Gamma_{t+1}-\mu G_{t+1})\right] - \tilde{R}_{e,t}^{I,a}}_{=-\frac{\tilde{R}_{e,t}^{I,a}}{\kappa_t}}\right\} \\
&(\bar{\omega}_{t+1}) && 0 = \gamma NW_t\mathbb{E}_e\left[(-\Gamma_{\omega,t+1})R_{t+1}'^{k}\right] + \lambda_t^E\mathbb{E}_t R_{t+1}'^{k}\left(\Gamma_{\omega,t+1} - \mu G_{\omega,t+1}\right)
\end{aligned}
$$

Eliminate the multiplier λ_t^E, the optimality conditions of this problem boil down to

$$\mathbb{E}_t\left[(1-\Gamma_{t+1})R_{t+1}'^{k}\right] = \frac{\tilde{R}_{e,t}^{I,a}}{\kappa_t}\left[\frac{\mathbb{E}_t\left(\Gamma_{\omega,t+1}R_{t+1}'^{k}\right)}{\mathbb{E}_t\left[\left(\Gamma_{\omega,t+1} - \mu G_{\omega,t+1}\right)R_{t+1}'^{k}\right]}\right].$$

Utilization Rate. As the main text indicates, the entrepreneur's optimization problem choosing the utilization rate is

$$\max_{u_t} r_t^k K_{u,t} - \Psi_k(u_t)K_{t-1}.$$

and the optimality condition is

$$r_t^k = \Psi_k'(u_t)$$

A.4 Financial Intermediary Optimality

The sum of the two portfolio constraints amounts to a standard balance sheet constraint: $M_t + B_t \leq D'_t + K^b_t$. Using this balance sheet constraint twice at t and $t+1$, the FI's objective function at t becomes[3]

$$\max_{D_t, B_t, D'_t, s_t} \quad \mathbb{E}_t \Bigg[\left(\tilde{R}^{I,a}_{m,t} - 1\right) M_t + \left(\tilde{R}^{I,a}_{e,t} - 1\right) B_t - \left(\tilde{R}'_t - 1\right) D'_t \\ - \Theta_t D_t - \Xi_t B_t - \frac{\phi_b}{2}\left(\frac{K^b_t}{S_t} - \nu_b\right)^2 K^b_t - acc_{s,t} \Bigg]$$

subject to the portfolio constraints

$$M_t \leq s_t(D'_t + K^b_t),$$
$$B_t \leq (1 - s_t)(D'_t + K^b_t).$$

Let λ^M_t and λ^B_t be the multipliers on the two constraints respectively. The FONCs are

$$\begin{aligned}
(D_t) \qquad & \lambda^M_t = \tilde{r}^{I,a}_{m,t} - \Theta_t + \phi_b \left(\frac{K^b_t}{S_t} - \nu_b\right)\left(\frac{K^b_t}{S_t}\right)^2, \\
(B_t) \qquad & \lambda^B_t = \tilde{r}^{I,a}_{e,t} - \Xi_t + \phi_b \left(\frac{K^b_t}{S_t} - \nu_b\right)\left(\frac{K^b_t}{S_t}\right)^2, \\
(D'_t) \qquad & 0 = \tilde{r}'_t - \lambda^M_t s_t - \lambda^B_t (1 - s_t), \\
(s_t) \qquad & 0 = (\lambda^M_t - \lambda^B_t) S_t - \frac{1}{2}\Phi_{pm,t} M_t + \frac{1}{2}\Phi_{pe,t} B_t - \frac{\partial ac_{s,t}}{\partial s_t}.
\end{aligned}$$

Eliminating the multipliers leads to following result

$$\begin{aligned}
(D'_t) \qquad & \tilde{r}^{I,a}_{m,t} s_t + \tilde{r}^{I,a}_{e,t}(1 - s_t) \\
& = \tilde{r}'_t + \underbrace{\frac{1}{2}\Phi_{p,t} s_t^2 + \frac{1}{2}\Phi_{pe,t}(1 - s_t)^2 - \phi_b \left(\frac{K^b_t}{S_t} - \nu_b\right)\left(\frac{K^b_t}{S_t}\right)^2}_{\text{Interest Premium}}; \\
(s_t) \qquad & s_t = \frac{1}{\Phi_{p,t} + \Phi_{pe,t}} \left(\tilde{r}^{I,a}_{m}, t - \tilde{r}^{I,a}_{e,t} + \Phi_{pe,t} - \frac{\partial acc_{s,t}}{\partial s_t}/S_t\right).
\end{aligned}$$

where $\frac{\partial acc_{s,t}}{\partial s_t}/S_t = \phi_s(s_t - s_{t-1})$.

The Mortgage Share. The discussion on this point can be found in the main text.

[3]Note that the problem now is essentially intra-temporal.

The Interest Rate Spreads. The interest premia are related to the portfolio share in this model. Substitute away one interest rate in the FONC of D_t', one can show that

$$\begin{aligned}
\tilde{r}_{m,t}^{I,a} - \tilde{r}_t' &= \Phi_{pm,t} s_t (1-s_t) + \frac{1}{2}\Phi_{pm,t} s_t^2 - \frac{1}{2}\Phi_{pe,t}(1-s_t)^2 \\
&\quad + \phi_s (s_t - s_{t-1})(1-s_t) - \phi_b \left(\frac{K_t^b}{S_t} - \nu_b\right)\left(\frac{K_t^b}{S_t}\right)^2; \\
\tilde{r}_{e,t}^{I,a} - \tilde{r}_t' &= \Phi_{pe,t} s_t (1-s_t) + \frac{1}{2}\Phi_{pe,t}(1-s_t)^2 - \frac{1}{2}\Phi_{pm,t} s_t^2 \\
&\quad - \phi_s (s_t - s_{t-1}) s_t - \phi_b \left(\frac{K_t^b}{S_t} - \nu_b\right)\left(\frac{K_t^b}{S_t}\right)^2.
\end{aligned}$$

A.5 Production Sector Optimality

The Intermediate Good Producer. The intermediate good firm is subject to the intra-temporal cost minimization problem below:

$$\begin{aligned}
\min_{N_s', Nt, K_{u,t}} &\left(w_t' N_t' + w_t N_t + r_t^k K_{u,t}\right) - \frac{P_t^w}{P_t} Y_t \\
s.t. \quad & Y_t \le A_t \left(N_t'^{\alpha} N_t^{1-\alpha}\right)^{1-\mu_c} K_{t-1}^{\mu_c}
\end{aligned}$$

where P_t^w stands for the whole sale price of the intermediate good. It is deflated by the final good price index P_t so that the cost minimization problem is consistent in real term. The FONCs are:

$$\begin{aligned}
(N_t') \quad w_t &= \frac{1}{X_t} \cdot (1-\mu_c)\alpha \frac{Y_t}{N_t'} \\
(N_t) \quad w_t &= \frac{1}{X_t} \cdot (1-\mu_c)(1-\alpha) \frac{Y_t}{N_t} \\
(K_{u,t}) \quad r_t^k &= \frac{1}{X_t} \cdot \mu_c \frac{Y_t}{K_{u,t}}
\end{aligned}$$

where I denote the $X_t = \frac{P_t}{P_t^w}$ as the average mark-up of the final good price level to the intermediate good whole sale price. The term $\frac{1}{X_t}$ can be interpreted as the average marginal cost for the retailers. I define $mc_t = \frac{1}{X_t}$ for later use.

Retailers. An individual retailer transforms the homogeneous intermediate good to its type of final retailing good at no cost. It is like a labeling or branding procedure in reality. For aggregation convenience let these retailers take address on the unit interval and indexed by $i \in [0,1]$. Since the retail good market is monopolistic competitive, the retailer i can set its price as $P_t(i)$. The quantity of its product $Y_t(i)$ is co-determined with its price, for that the retailer is subject to its demand curve.

Let there be a final aggregator who assembles all retailers' output into one final good Y_t^f using a constant elasticity of substitution (CES) aggregation technology. It's (real) cost minimization problem is

$$\min_{Y_t(i)} \frac{1}{P_t}\left(\int_0^1 P_t(i)Y_t(i)\mathrm{d}i - P_t Y_t^f\right)$$

$$s.t.\quad Y_t^f \le \left(\int_0^1 Y_t(i)^{\frac{\varepsilon-1}{\varepsilon}}\right)^{\frac{\varepsilon}{\varepsilon-1}}$$

where the parameter $\varepsilon > 1$ is the individual demand curve elasticity.The FONCs are

$$\frac{P_t(i)}{P_t} = {Y_t^f}^{\frac{1}{\varepsilon}} Y_t(i)^{-\frac{1}{\varepsilon}}$$

$$Y_t^f = \left(\int_0^1 Y_t(i)^{\frac{\varepsilon-1}{\varepsilon}}\right)^{\frac{\varepsilon}{\varepsilon-1}}$$

The first FONC gives the individual demand curve the retailer i faces, i.e.

$$P_t(i) = \left(\frac{Y_t(i)}{Y_t^f}\right)^{-\frac{1}{\varepsilon}} P_t, \text{ or } Y_t(i) = \left(\frac{P_t(i)}{P_t}\right)^{-\varepsilon} Y_t^f$$

Following these results, the price index expression is

$$P_t = \left(\int_0^1 \left(\frac{P_t(i)}{P_t}\right)^{1-\varepsilon}\right)^{\frac{1}{1-\varepsilon}}.$$

Retailers adjusts their price subject to the mechanism *ála Calvo.* The retailer i chooses its price $P_t(i)$ taking the intermediate wholesale price P_t^w, the price level P_t and the demand curve of its goods as given, and $P_t(i)$ can only be re-optimized in ever period with probability $1-\theta$. Otherwise, the price in this period will be reset according to a partial indexation rule, such that it goes up partly according to the previous period inflation rate with weight ι, while the other $1-\iota$ fraction goes up with the steady-state inflation rate $\bar{\pi}$.

The optimal price resetting problem for a representative retailer i that can do so at period t then becomes

$$\max_{P_t(i)} \mathbb{E}_t \sum_{k=0}^{\infty} \theta^k \Lambda_{t,t+k} \cdot \frac{1}{P_{t+k}} \left[\left(\prod_{s=1}^{k} \pi_{t-1+s}^{\iota} \bar{\pi}^{1-\iota}\right) P_t(i) Y_{t+k}(i) - P_{t+k}^w Y_{t+k}(i)\right]$$

subject to its demand curve in each period starting from t, i.e.

$$Y_{t+k}(i) = \left(\frac{P_{t+k}(i)}{P_{t+k}}\right)^{-\varepsilon} Y_{t+k}^f, \text{ for } k = 0, 1, 2, \ldots.$$

Here, $\Lambda_{t,t+k} \equiv \beta^k \mathbb{E}_t \frac{\lambda_{t+k}}{\lambda_t}$ is the household relevant stochastic discount factor where λ_t stands for the marginal utility of household at period t. Also, for notation convenience, denote $\tilde{\pi}_{t,s} = \pi_{t-1+s}^{\iota} \bar{\pi}^{1-\iota}$ which is the price indexation term applied after s period passed since t, i.e. $t+s$.

The FONC with respect to $P_t(i)$ is

$$0 = \mathbb{E}_t \sum_{k=0}^{\infty} \theta^k \Lambda_{t,t+k} Y_{t+k}^f \left[(1-\varepsilon)\left(\frac{\prod_{s=1}^k \tilde{\pi}_{t,s} P_t^*(i)}{P_{t+k}}\right)^{-\varepsilon} \left(\frac{\prod_{s=1}^k \tilde{\pi}_{t,s}}{P_{t+k}}\right) - \frac{-\varepsilon}{X_{t+k}} \left(\frac{\prod_{s=1}^k \tilde{\pi}_{t,s} P_t^*(i)}{P_{t+k}}\right)^{-\varepsilon-1} \left(\frac{\prod_{s=1}^k \tilde{\pi}_{t,s}}{P_{t+k}}\right)\right].$$

With some algebra, one can get

$$0 = \mathbb{E}_t \sum_{k=0}^{\infty} \theta^k \Lambda_{t,t+k} Q^{\mathbb{1}}_{t+k} Y^f_{t+k} \left[\left(\prod_{s=1}^{k} \frac{\tilde{\pi}_{t,s}}{\pi_{t+s}} \right)^{1-\varepsilon} \left(\frac{P^*_t(i)}{P_t} \right) - \frac{\varepsilon}{\varepsilon - 1} mc_{t+k} \left(\prod_{s=1}^{k} \frac{\tilde{\pi}_{t,s}}{\pi_{t+s}} \right)^{-\varepsilon} \right].$$

By the symmetric setup, all retailers that can re-optimize will choose the same price. Thus $P^*_t(i) = P^*_t$. Denote $\pi^*_t = \frac{P^*_t}{P_t}$. To write the above FONC in a recursive format, define

$$g^1_t = \mathbb{E}_t \sum_{k=0}^{\infty} (\beta\theta)^k \lambda_{t+k} Y^f_{t+k} \left(\prod_{s=1}^{k} \frac{\tilde{\pi}_{t,s}}{\pi_{t+s}} \right)^{-\varepsilon} \cdot mc_{t+k};$$

$$g^2_t = \mathbb{E}_t \sum_{k=0}^{\infty} (\beta\theta)^k \lambda_{t+k} Y^f_{t+k} \left(\prod_{s=1}^{k} \frac{\tilde{\pi}_{t,s}}{\pi_{t+s}} \right)^{1-\varepsilon} \cdot \pi^*_t.$$

and from FONC it is obvious that

$$g^2_t = \frac{\varepsilon}{\varepsilon - 1} g^1_t,$$

and the following two equations hold:

$$g^1_t = \lambda_t Y^f_t mc_t + \beta\theta \mathbb{E}_t \left(\frac{\tilde{\pi}_{t,1}}{\pi_{t+1}} \right)^{-\varepsilon} \cdot g^1_{t+1};$$

$$g^2_t = \lambda_t Y^f_t \pi^*_t + \beta\theta \mathbb{E}_t \left(\frac{\tilde{\pi}_{t,1}}{\pi_{t+1}} \right)^{1-\varepsilon} \cdot \left(\frac{\pi^*}{\pi^*_{t+1}} \right) \cdot g^2_{t+1}.$$

From the final good price index, the following relationship can be derived[4]

$$1 = \theta \left(\frac{\pi^{\iota}_{t-1} \bar{\pi}^{1-\iota}}{\pi_t} \right)^{1-\varepsilon} + (1-\theta) \pi^{*\,1-\varepsilon}_t.$$

Log-linearizing these results leads to the New Keynesian Phillips curve in the main text.

Capital Producer. Following the maximization problem in the main text, the FONC is

$$q^k_t = 1 + \frac{\phi_k}{2} \left(\frac{I_t}{I_{t-1}} - 1 \right)^2 + \phi_k \left(\frac{I_t}{I_{t-1}} - 1 \right) \frac{I_t}{I_{t-1}} - \mathbb{E}\Lambda_{t,t+1} \phi_k \left(\frac{I_t}{I_{t-1}} - 1 \right) \left(\frac{I_t}{I_{t-1}} \right)^2.$$

This completes all optimality conditions in the model.

[4] There is also a price dispersion term derived in this part, which plays no role under first-order approximation.

B Data Description

The model is estimated with U.S. data at quarterly frequency. Data with secular trends are detrended using quadratic detrending and then divided by four to change to the quarterly unit following the model. Data with no trend are demeaned, for instance, inflation rate.

B.1 Definition of data variables (observables)

Output: ln(A939RX0Q048SBEA).
Consumption: ln(A939RX0Q048SBEA).
Non-residential investment: ln(PNFI/GDPDEF/POPULATION_BEA×1000000×100).
Real house price: ln(CSUSHPISA/GDPDEF×100)
Inflation: ln(GDPDEF/GDPDEF(-1)).
Losses on business loans: CORBLACBS×(OLALBSNNB+BLNECLBSNNCB)/GDP.
Losses on mortgage loans: CORSFRMACBS×HHMSDODNS/GDP.
FFR: FEDFUNDS/4
Prime mortgage spread: (MORTGAGE30US−GS10)/4.
AAA spread: AAA10Y/4.
Prepayment rate: CPR/4
Share of mortgage in asset: HHMSDODNS/(HHMSDODNS+OLALBSNNB+BLNECLBSNNCB).

Also, I define a population rate series as below:[5]:
POPULATION_BEA = PCECC96/A794RX0Q048SBEA×1000000 (unit: thousand people).

B.2 Data and sources

This section lists the name and source of the data. Other than the conditional prepayment rate, the data are retrieved from FRED, Federal Reserve Bank of St. Louis.

A939RX0Q048SBEA: Real gross domestic product per capita - Chained 2012 Dollars, Seasonally Adjusted Annual Rate.
Source: U.S. Bureau of Economic Analysis.

A939RX0Q048SBEA: Real gross domestic product per capita - Chained 2012 Dollars, Seasonally Adjusted Annual Rate.
Source: U.S. Bureau of Economic Analysis.

PCECC96: Real Personal Consumption expenditure - Billions of Chained 2012 Dollars, Seasonally Adjusted Annual Rate.
Source: U.S. Bureau of Economic Analysis.

PNFI: Private Nonresidential Fixed Investment - Billions of Dollars, Seasonally Adjusted Annual Rate.
Source: U.S. Bureau of Economic Analysis.

GDPDEF: Gross Domestic Product: Implicit Price Deflator - Index 2012=100, Seasonally Adjusted.
Source: U.S. Bureau of Economic Analysis.

[5] This population rate is used to calculate the real non-residential investment per capita as seen above. Using this constructed series guarantees consistency in the data definition used by the U.S. Bureau of Economic Analysis.

CSUSHPISA: S&P\Case-Shiller U.S. National Home Price Index - Index Jan 2000=100, Seasonally Adjusted.
S&P Dow Jones Indices LLC.

PCECC96: Real Personal Consumption expenditure, Billions of Chained 2012 Dollars, Seasonally Adjusted Annual Rate.

CORSFRMACBS: Charge-Off Rate on Single Family Residential Mortgages, Booked in Domestic Offices, All Commercial Banks - Percent, Seasonally Adjusted.
Source: Board of Governors of the Federal Reserve System (US).

HHMSDODNS: Households and Nonprofit Organizations; Home Mortgages; Liability, Level - Billions of Dollars, Seasonally Adjusted. Annual Rate.
Source: Board of Governors of the Federal Reserve System (US).

CORBLACBS: Charge-Off Rate on Commercial and Industrial Loans, All Commercial Banks - Percent, Seasonally Adjusted.
Source: Board of Governors of the Federal Reserve System (US).

OLALBSNNB: Nonfinancial Noncorporate Business; Other Loans and Advances; Liability, Level - Billions of Dollars, Not Seasonally Adjusted. Annual Rate.
Source: Board of Governors of the Federal Reserve System (US).

BLNECLBSNNCB: Nonfinancial Corporate Business; Depository Institution Loans Not Elsewhere Classified; Liability, Level - Billions of Dollars, Not Seasonally Adjusted. Annual Rate.
Source: Board of Governors of the Federal Reserve System (US).

GDP: Gross Domestic Product- Billions of Dollars - Seasonally Adjusted Annual Rate.
Source: U.S. Bureau of Economic Analysis.

FEDFUNDS: Effective Federal Funds Rate - Percent, Not Seasonally Adjusted.
Source: Board of Governors of the Federal Reserve System (US).

MORTGAGE30US: 30-Year Fixed Rate Mortgage Average in the United States - Percent, Not Seasonally Adjusted. Source: Freddie Mac.

GS10: 10-Year Treasury Constant Maturity Rate - Percent, Not Seasonally Adjusted.
Source: Board of Governors of the Federal Reserve System (US).

AAA10Y: Moody's Seasoned Aaa Corporate Bond Yield Relative to Yield on 10-Year Treasury Constant Maturity - Percent, Not Seasonally Adjusted.
Source: Federal Reserve Bank of St. Louis.

CPR: FNM30 MBS conditional prepayment rate. Annual Rate.
Source: eMBS.