

# Monetary Policy and Credit Supply Adjustment with Endogenous Default and Prepayment\*

## Job Market Paper

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### Abstract

This paper develops a general-equilibrium model to study how the credit supply mechanisms in the financial intermediation sector, which lends to households and entrepreneurs subject to financial frictions, influence monetary policy. In the model, endogenous default of mortgage and business loans and prepayment of household mortgages influence the costs of supplying credit from the financial intermediary (FI). The FI optimizes its loan portfolio composition given these cost variations with frictions. The loan contracts between the FI and borrowers allow these two parties to share aggregate risk, deviating from the canonical work by [Bernanke et al. \(1999\)](#). I estimate the model with U.S. data. Likelihood inference indicates positive credit supply cost elasticities, significant frictions to portfolio adjustment, and balance-sheet strength fluctuation to borrowers' default and prepayment variations. Given households' endogenous behaviors, conventional monetary policy's effectiveness in stabilizing inflation is enhanced under a TFP shock but reduced under a mortgage loan risk shock, and the credit supply channels worsen the latter situation. The effectiveness of unconventional monetary policy is enhanced by the credit supply channel.

*JEL Classification:* E32, E44, E52, G21.

*Keywords:* DSGE model, Financial intermediation, Financial frictions, Housing, Bayesian estimation, Monetary policy.

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# 1 Introduction

Since the latest crisis in the U.S., a large amount of research has been devoted to understanding the fluctuation of the business cycle caused by financial frictions and the propagation mechanisms of adverse shocks originating in the housing and financial sectors given these frictions.<sup>1</sup> The extent to which activities of borrowers (households and entrepreneurs), such as defaulting on liabilities or refinancing, disrupt the intermediation process is a major concern. Also, how the monetary policy interventions are influenced by these mechanisms is crucial for understanding the effectiveness of policy.

This paper studies the credit supply mechanisms in the financial intermediation sector, given borrowers face various frictions. Specifically, it examines how the financial intermediary's credit supply costs, net worth and asset portfolio composition vary with borrowers' endogenous default and prepayment activities, and if these mechanisms have significant aggregate implications. The paper also investigates how channels affect the transmission of monetary policy interventions.

To do so, the paper first estimates with Bayesian methods and U.S. data a DSGE model. Results regarding the monetary policy implications based on the estimated model include the following. For conventional monetary policy, stabilizing inflation can be more difficult to achieve given the disturbance from the mortgage market.<sup>2</sup> I further find that the credit supply channels introduced in this paper exacerbate this inefficiency issue. For unconventional monetary policy considered in this paper, its effectiveness is enhanced by the mortgage credit channel and the credit supply channels.

In reality, it is costly for a lender to offer credit to borrowers (households and entrepreneurs), since borrowers can default on their debt obligations, causing losses to a financial intermediary.<sup>3</sup> In addition, households' mortgage debt can be refinanced, which incurs a managerial cost in the intermediation process.<sup>4</sup> These actions bring variations to the financial intermediary's net worth and interest rate spreads over the business cycle, potentially explaining the fluctuations shown in figure 1. How credit supply costs are influenced by these activities is a central focus of this paper.

Also as seen in figure 2, the liability composition carried by the private sector varies

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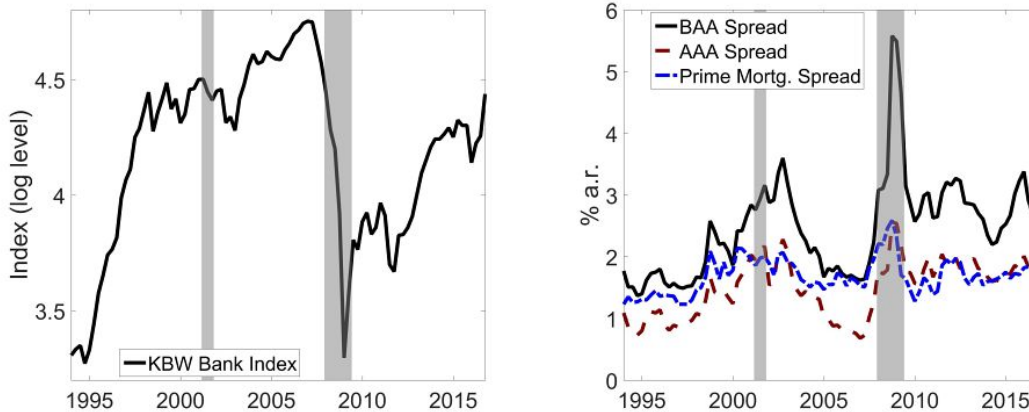
<sup>1</sup> One branch of research focuses more on the housing sector, see [Iacoviello and Neri \(2010\)](#), [Elenev \(2017\)](#), or [Greenwald \(2018\)](#); while some other studies focus more on explaining the issue in the financial intermediation sector, such as [Gerali et al. \(2010\)](#), [Curdia and Woodford \(2011\)](#), [Gertler and Karadi \(2011\)](#), [Landvoigt \(2016\)](#). Recent studies such as [Landvoigt \(2016\)](#) and [Ferrante \(2018\)](#) emphasize both features in their frameworks. These studies also investigate the policy interventions of the fiscal and monetary authorities. See [Del Negro et al. \(2017\)](#) for a concise survey of this literature.

<sup>2</sup>This result works through the mortgage credit channel (explained later) introduced by [Greenwald \(2018\)](#) and complements his finding that stabilizing inflation is easier to achieve with this channel under a deflationary TFP shock.

<sup>3</sup>The Great Recession is a recent and extreme example. As the housing bubble burst in 2006, homeowners started defaulting on mortgage payments greatly in 2007. These loan losses are considered responsible for triggering the financial sector meltdown, such as the fall of Lehman Brothers in 2008.

<sup>4</sup>See the introduction of the Prepayment Monitoring Report by FHFA for a brief explanation of why the mortgage prepayment rate variation in the financial market incurs potential costs to investors: [https://www.fhfa.gov/AboutUs/Reports/ReportDocuments/Prepayment-Monitoring\\_2Q2018.pdf](https://www.fhfa.gov/AboutUs/Reports/ReportDocuments/Prepayment-Monitoring_2Q2018.pdf).

Figure 1: Financial Sector Net Worth and Interest Rate Spreads



*Notes:* The left panel shows the KBW bank index in log level as a proxy measure of the financial sector net worth. The right panel plots three interest rate spreads between the Yield on 10-Year Treasury Constant Maturity and the following three interest rates: the Moody’s Seasoned Baa and Aaa Corporate Bond Yields (labelled BAA Spread and AAA spread respectively) and the 30-Year Fixed Rate Mortgage Average (labelled Prime Mortg. Spread). Shaded bars represent NBER recession periods.

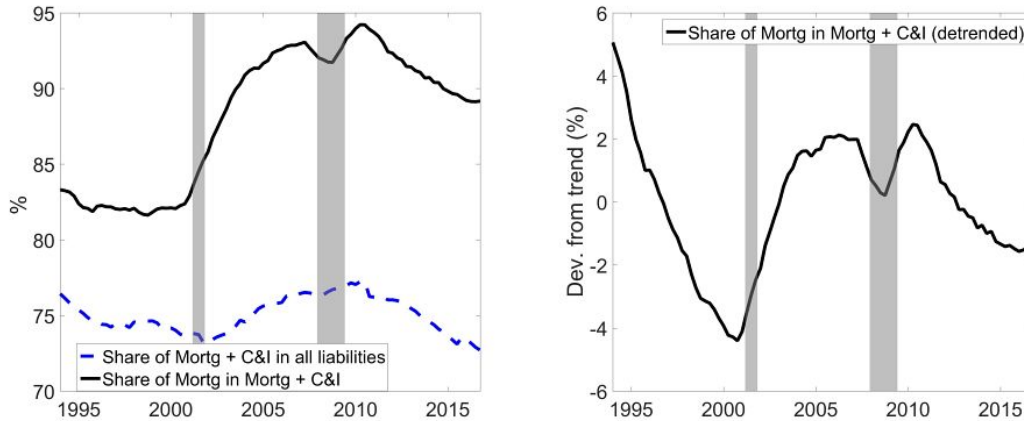
overtime. Given demand-side factors that are important determinants of the equilibrium quantities and composition of credit observed in the plot, it is natural to wonder about the role of supply-side factors, which are determined by the financial intermediation sector in the economy. By choosing its portfolio composition, the financial intermediary substitutes away from assets suffering a supply cost increase to those supplied at lower cost. This can put upward pressure on the price of credit and suppress the equilibrium quantity further, deepening the impact of the adverse shocks. However, relatively less is known about supply-side effects as few related studies have been conducted.<sup>5</sup>

In order to answer these questions, this paper develops a dynamic stochastic general equilibrium model with endogenous default, prepayment, intermediary loss and portfolio choice. The intermediation costs of the lender (financial intermediary) supplying loans vary with the borrowers’ endogenous default and prepayment decisions. The lender also takes losses when more defaults happen than expected, weakening its balance sheet strength and thus influencing the supply of credit indirectly. Finally, yet importantly, the financial intermediary optimally chooses the composition of assets it holds on its balance sheet. These activities are quantified by estimating the model.

The structural model consists of five sectors: households, entrepreneurs, financial intermediary, production and monetary authority. Patient and impatient households popu-

<sup>5</sup>Den Haan et al. (2007) investigate the portfolio behavior of bank loans following a monetary tightening and show the presence of the supply-side effect. Orzechowski (2017) finds empirical evidence of portfolio shifts at banks under monetary policy shocks. Dib (2010), Bae (2012), and more recently Giri (2018) feature the portfolio choice of the financial intermediary in the interbank market, risky entrepreneur loans and riskless government bonds, and discuss the implication to the interbank market friction and ‘flight to quality’ phenomenon seen in the recession. However, the trade-off between different private loans is rarely seen in general equilibrium studies.

Figure 2: Liability Composition in the Private Sector



*Notes:* The blue dashed line is the share of the residential mortgage loan and C&I loan in total private sector liabilities. The black line on the left panel is the share of the residential mortgage loan in the C&I and residential mortgage loan pool. The black line on the right panel is the deviation of this share from its quadratic trend. Shaded bars represent NBER recession periods.

late the household sector as in [Iacoviello \(2005\)](#). Impatient households borrow long-term mortgage loans that are defaultable and prepayable. These features are modeled following related works in the literature such as [Forlati and Lambertini \(2011\)](#), [Elenev \(2017\)](#), and [Greenwald \(2018\)](#). Entrepreneurs, as the other group of borrowers in the economy, carry one-period defaultable debt following [Bernanke et al. \(1999\)](#). Market incompleteness emerges in financial contracts between the financial intermediary (lender) and these two kinds of borrowers such that the contracts are signed based on expected returns. Such a setup is in line with [Zhang \(2009\)](#) and [Jakab and Kumhof \(2015\)](#), in which the ex-post return can deviate from expectations due to aggregated shocks, and therefore can impose losses on the lender. The financial intermediary also optimizes its portfolio composition with frictions, with the relevant model feature similar to works by [Bae \(2012\)](#) and [Dib \(2010\)](#) among others. Other sectors in the model are relatively standard.

The contribution and findings of this paper are three-fold. First, it gives a rich modeling environment of the household sector featuring endogenized mortgage loan default and prepayment in a monetary general equilibrium model.<sup>6</sup> It also incorporates entrepreneurs who can endogenously default on their loans. As stated, these borrowers' endogenized activities influence the supply costs of the financial intermediary as well as the outcome of policy intervention. I show that the conventional monetary policy effect is amplified under a deflationary TFP shock because the reaction of households to interest cuts through these margins is stronger.<sup>7</sup> However, the effectiveness is dampened when a mort-

<sup>6</sup>The combination of these features is relatively rare in the literature. In recent studies, [Elenev \(2017\)](#) is one that is similar to the framework in this paper. Different from the current paper, his work does not consider business loans in the economy and focuses on policy involving large-scale asset purchases.

<sup>7</sup> A side effect though, is that this mechanism may create a faster increase in borrowers' leverage when the interest rate is lowered to stimulate the economy, as in the policy dilemma discussed in [Greenwald](#)

gage risk shock hits. This result is due to a larger contraction of household debt through the default margin, changing the transmission of monetary policy and indicating potential needs for alternative policy substitutes. I also study an unconventional monetary policy lowering the long-term interest rate, whose effect depends largely on households' endogenized default. These results suggest that the monetary policy prospects can differ greatly given different macroeconomic scenarios.

Second, the paper links the loss for the financial intermediary to the real sector's default activities. In the classical framework *à la* [Bernanke et al. \(1999\)](#), the form of the loan contract immunizes the lender from taking a loss. This paper sets a novel contract form that only allows for an expected return rate for the financial intermediary by assumption; therefore, the aggregate risk is shared between the financial intermediary and borrowers, although the former can diversify the idiosyncratic risks on the asset side.<sup>8</sup> This contract form can make the agent's default impact spill over to the financial sector and decrease its net worth, and this linkage can form a vicious cycle, as the financial intermediary further decreases the credit supply to the real sector. As to the result, the proposed losses can explain significant portions of observed loan charge-offs by the financial intermediary. Nevertheless, these losses along with other endogenous frictions in the model do not perfectly account for the complete dynamic of net worth fluctuation in the financial sector observed in [Figure 1](#), albeit they can explain part of the movements according to model simulations shown later. Their impact over fluctuations of real variables including consumption and output are also weak for the chosen sample periods, needing other shocks to jointly explain the dynamics of these variables. In this vein, the paper supports the finding by [Suh and Walker \(2016\)](#) such that financial frictions have limited ability in explaining the financial crisis in a linearized, estimated model framework.<sup>9</sup>

The paper also looks into the portfolio choice activity by the financial intermediary. In the model, the financial intermediary supplies loans to borrowers with intermediation costs (heretofore called the 'intermediation cost mechanism').<sup>10</sup> Given these costs, the intermediary can optimally choose between the loans to the households or the entrepreneurs, subject to a portfolio adjustment cost (heretofore called the 'portfolio choice mechanism').<sup>11</sup> I find significantly positive estimates for the elasticities of credit supply costs to real sector activities. The estimate of the portfolio adjustment cost parameter is

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(2018).

<sup>8</sup>This point relating to default and redistribution between the borrowers and the financial intermediary is also discussed in [Zhang \(2009\)](#), [Quint and Rabanal \(2014\)](#), and [Jakab and Kumhof \(2015\)](#), among others. These studies feature contracts that can generate losses to the lender, sharing the same features as this paper.

<sup>9</sup>This result does not necessarily mean that shocks and frictions should not be a concern to the monetary authority. As stated later, this paper does not explicitly model the agency problem in the banking sector, which is the key mechanism amplifying borrowers' frictions and the culprit of the last recession. See [Paixao \(2018\)](#) for a model with such amplifications. Incorporating this mechanism is left for future research.

<sup>10</sup>These credit supply costs require the intermediary to consume real resources, similar to [Curdia and Woodford \(2011\)](#), for instance.

<sup>11</sup>As seen in [figure 2](#), the sum of commercial and industrial loans and residential mortgage loans takes a significant portion of the total private sector liability (around 75%), and this share fluctuates little over time starting from the early 90s. Without loss of generality, this paper takes these two loans as the only two private sector assets the financial intermediary can hold in the model.

positive yet moderate, supporting the view that the financial sector does actively change its asset positions subject to a certain degree of frictions. In the monetary policy analysis section, I further show that these credit supply mechanisms are crucial for policy outcomes. Specifically for the conventional monetary policy, deeper interest rate cuts may be needed to stabilize inflation under a mortgage loan risk shock, given the intermediation costs varying with real sector activities; for the unconventional monetary policy, the long-term rate (mortgage loan rate in this study) cut is effective only when the credit supply costs are lowered through the intermediation cost mechanisms. As to the portfolio choice mechanism, its influence on the conventional monetary policy outcome is quantitatively similar to the intermediation cost mechanism, and it marginally enhances the efficiency of the unconventional monetary policy. Overall, the analysis indicates the financial sector's importance for monetary policy pass-through, and the sensitivity of credit supply costs variation with the real sector activities is essential to be gauged. The conduct of monetary policy can be inefficient without significantly influencing the credit supply costs of the financial sector, who eventually lends credits to the rest of the economy.

## Related literature

This paper relates to a vast literature studying financial frictions in macroeconomics. Earlier seminal works include [Kiyotaki and Moore \(1997\)](#) (KM hereafter), [Bernanke et al. \(1999\)](#) (BGG hereafter), among others. Using different frameworks, these papers emphasize the same idea that financial market frictions can propagate and amplify various shocks to the whole economy, thus having a so-called 'financial accelerator' effect. The financial-friction theories also have been applied to the housing market literature. Early works include [Aoki et al. \(2004\)](#) and [Iacoviello \(2005\)](#). This literature bloomed after the Great Recession, due to its link to the housing market and the financial sector. One branch of this literature focuses on the frictions in the housing market and tries to explain the origination and mechanism causing the crisis. Related studies include [Forlati and Lambertini \(2011\)](#), [Iacoviello and Neri \(2010\)](#), and [Quint and Rabanal \(2014\)](#). These models incorporate a financial accelerator mechanism *à la* KM or BGG, and generally, their quantitative results explain the primary features of the recent Great Recession.

Another strand of the literature related to this paper focuses on banking sector imperfections and their impact during the recession. These works, including [Gertler and Karadi \(2011\)](#), [Farhi and Tirole \(2012\)](#), and [Ferrante \(2018\)](#), focus on the endogenous behavior of the financial intermediary during the recession and its consequences for credit supply and policy interventions. In a more simplified manner, [Gerali et al. \(2010\)](#) estimate a structural model with frictions in the retailing and wholesale banking sectors, incorporating a monopolistic market structure and costly bank net worth adjustment. I adopt their wholesale banking sector model straightforwardly, focusing on the balance sheet related effect on the financial intermediary's asset side, including its trade-off between different private assets.<sup>12</sup> Different from most of these works, I make the lender suffer potential losses

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<sup>12</sup>Therefore, the model does not incorporate bank runs which are considered a major cause of the last

when more default happens in the real sector by assuming that contracts are signed with expected returns ex-ante.<sup>13</sup> This linkage is different from the classical BGG setup where the lender never suffers losses. In this vein, my framework shares similarities with [Zhang \(2009\)](#), [Quint and Rabanal \(2014\)](#), and [Jakab and Kumhof \(2015\)](#), among others, regarding the endogenous losses on a financial intermediary's lending contract. Also, I quantify these losses by estimating the model.

Moreover, I include the portfolio choice for the financial intermediary between different private assets, which is a novel feature in financial intermediation models. This setup of variable costs supplying loans builds on the work by [Bae \(2012\)](#) and can be traced back to [Dib \(2010\)](#), [Curdia and Woodford \(2011\)](#), [Goodfriend and McCallum \(2007\)](#), and [Cook \(1999\)](#). These studies introduce mechanisms of the financial intermediary supplying loans to the real sector with costs influenced by variables in the economy. Among them, [Dib \(2010\)](#) models the bank's allocation of resources between interbank and other loans; [Bae \(2012\)](#) studies the portfolio choice of the bank in a recession between a safe government bond and risky business loans, commonly known as 'flight to quality.' Different from the above studies, this paper focuses on the portfolio choice behavior between household and business loans on the asset side of the balance sheet in the intermediation sector, which are two of the most important assets not only regarding quantity, but also for their roles in the latest recession.<sup>14</sup>

General equilibrium models are often utilized for evaluating monetary policies, especially unconventional ones used to remedy the last crisis. These works include [Gertler and Karadi \(2011\)](#) and [Curdia and Woodford \(2011\)](#), among others. More recent studies relating to the housing sector also address monetary policy related issues, with more realistic features of mortgage debt included, such as the long-term and the prepayment properties: see [Garriga et al. \(2017\)](#), [Pietrunti and Signoretti \(2017\)](#), [Landvoigt \(2016\)](#), [Elenev \(2017\)](#), [Ferrante \(2018\)](#), and [Greenwald \(2018\)](#), among others. These frameworks share many similarities to mine, with the last three papers being most closely related. With respect to [Ferrante \(2018\)](#), this paper differs in that the household has a chance to refinance its long-term mortgage, which can potentially bring a managerial cost to the financial intermediary in the framework presented by this paper. Also, the lender's cost is linked to borrowers' default and prepayment activities other than independent shocks to the financial intermediary that intensify its agency issues. In [Elenev \(2017\)](#), the households have prepayable and defaultable long-term debt similar to my framework, and the model analyzes large-scale asset purchase policies. The focus of this paper, on the other hand, is the interaction between the lender and borrowers, and the lender's trade-off between supplying household and business loans, as well as the potential impact of these frictions to the propagation of exogenous disturbances. The model's household block is built on the

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financial crisis; see [Brunnermeier and Sannikov \(2014\)](#) for instance.

<sup>13</sup>The loss in the banking sector has been studied and modeled using an agnostic or exogenous approach in past studies, e.g. [Gerali et al. \(2010\)](#) and [Iacoviello \(2015\)](#).

<sup>14</sup>The study shares similarities with recent work by [Aksoy and Basso \(2014\)](#) which studies the impact of bank's portfolio decision influencing term spreads in a theoretical model using a higher order approximation. I also show the impact of portfolio choice over interest rate spreads in the appendix.

work by [Greenwald \(2018\)](#), which also focuses on explaining the housing sector behavior, including the prepayment activity and the aggregate loan-to-value movement, as well as the interaction between households and the monetary authority. This paper complements Greenwald's work by adding a defaultable feature for impatient households, and further explores the effectiveness of monetary policies under the mortgage loan risk shock. The role of credit supply mechanisms proposed by this paper to the monetary policy transmission is also discussed.

The remainder of the paper is organized as follows. Section 2 presents the model. Section 3 discusses the calibration and estimation of the model. Section 4 exploits the model mechanisms, including model validation, source of fluctuations, decompositions of prepayment and default incentives, portfolio mechanism demonstration, and liquidity shock hypothesis check. Section 5 conducts experiments about conventional and unconventional monetary policies. Section 6 concludes.

## 2 The Model

The model contains multiple sectors. To introduce heterogeneity in the household sector to create a loan market, I follow the literature and assume that there are impatient and patient households that differ in their discount rate. There is also a business sector populated by agents called entrepreneurs, each endowed with a business project at the beginning of each period, who need external funds to run projects due to their insufficient net worth to be self-financed.

There is one financial intermediary in the model economy.<sup>15</sup> It is the only institution funneling loanable funds from patient households, who lend to the intermediary in the form of deposits, to the impatient households and entrepreneurs, who finance housing purchases and business projects respectively. The production sector contains a group of capital producers who refurbish capital each period for the use of entrepreneurs' projects; a constant returns-to-scale intermediate good producer who rents the capital stock from entrepreneurs and employs labor from households; and a group of retailers who differentiate the intermediate goods. These differentiated retailing goods are then aggregated to form the final consumption good. The government sector includes a monetary authority setting interest rate policy based on a Taylor-type rule. It also can perform unconventional monetary policy by altering the newly-issued mortgage loan rate discussed below.

### 2.1 Households

The economy consists of two kinds of households, each populated by a continuum of infinitely-lived agents. The two household groups differ in their preference: the impatient households have a smaller discount factor than the patient ones, i.e.,  $\beta < \beta'$ . For general notation, the variables and parameters for the two kinds of households are largely denoted by the same letters with superscript prime distinguishing those for savers. The

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<sup>15</sup>See [Williamson \(1987\)](#) for a discussion of why large-scale intermediation can endogenously emerge.



population weights for the impatient and patient households are denoted by parameters  $\gamma_h$  and  $\gamma_s$  respectively. I assume a unit mass of the population in each group for aggregation convenience.

Within each cohort of households, agents have access to a complete asset market for consumption and housing services, which provides complete insurance against idiosyncratic risks within the cohort; however, there is no such asset market between these two groups. This setup makes it possible to write the optimization problem conveniently in a representative agent format.<sup>16</sup>

## Impatient Households

A representative agent in the impatient household group maximizes expected lifetime utility as follows

$$\mathbb{E}_t \sum_{k=0}^{\infty} \beta^k z_{t+k} \left\{ \log(c_{t+k} - \eta c_{t+k-1}) + j_t \log(h_{t+k}) - v \frac{n_{t+k}^{1+\chi}}{1+\chi} \right\} \quad (1)$$

where nondurable good consumption  $c_t$  (and its value in the previous period  $c_{t-1}$ ), housing stock  $h_t$  and labor supply  $n_t$  are the arguments of the utility function at period  $t$ . Parameter  $\eta \in [0, 1)$  measures external habits in consumption. Parameters  $v$  and  $\chi$  are the weight on work disutility and the inverse Frisch elasticity of labor supply, both of which are positive. To sum, the household gets utility from nondurable good consumption and housing stock, and disutility from working. The household receives the wage rate  $w_t$  as compensation for supplying labor and pays a proportional tax  $\tau_y$  of the labor income.

The term  $z_t$  captures a shock to intertemporal preference, while variable  $j_t$  is a shock varying housing preference exogenously.<sup>17</sup> They evolve according to the following processes:

$$\log z_t = \rho_z \log z_{t-1} + \epsilon_{z,t}; \quad (2)$$

$$\log j_t = \rho_j \log j_{t-1} + (1 - \rho_j) \log \bar{j} + \epsilon_{j,t}, \quad (3)$$

where  $\epsilon_{z,t}$  and  $\epsilon_{j,t}$  are independently and identically distributed (i.i.d.) processes with standard deviations  $\sigma_z$  and  $\sigma_j$ .

The impatient household borrows from the intermediary to finance housing services and consumption in the form of the mortgage contract, denoted by  $m_t$  as its stock at the end of period  $t$ . The mortgage asset is modeled as a perpetuity with fixed nominal coupon rate and geometrically decaying stock. In general, variables with superscript star (for example  $r_{m,t}^*$  or  $m_t^*$ ) denote those for newly originated loans, distinguishing variables from existing lending contracts in the economy.<sup>18</sup>

<sup>16</sup>In the main text, I directly use the representative agent description when possible, and a disaggregate version can be found in the appendix.

<sup>17</sup>I add these preference shocks following the Bayesian DSGE literature, such as [Jacoviello and Neri \(2010\)](#).

<sup>18</sup>This notation is necessary to keep track of variables under the long-term fix-rate mortgage in the model

By holding the mortgage debt, the impatient households face two types of idiosyncratic shocks each period: the home quality shock and the prepayment cost shock, with the former affecting the household's choice of mortgage default and the latter prepayment.<sup>19</sup> For the timing, I assume that the impatient households receive the housing quality shock prior to the prepayment cost shock, thus making the default decision first, and only households who have repaid their debt can choose whether to refinance their mortgage. This setup reflects that households have to honor their obligations before considering prepaying them.<sup>20</sup>

**Mortgage Default.** At the beginning of each period  $t$ , the impatient household indexed by  $i$  has to pay a maintenance cost proportional to the housing stock owned from last period denoted as  $\delta_h h_{t-1}$ .<sup>21</sup> Then, she receives a revaluation of her housing stock as  $\omega_{m,t}^i q_t^h h_{t-1}$ , where  $q_t^h$  denotes the real house price, and  $\omega_{m,t}^i$ , named 'the quality shock', is drawn independently across all borrower households from the same distribution, which is time varying and mean preserving (to unity), and has a non-negative support. Following related studies, I choose the distribution to be log-normal, and  $\mathbb{E}(\omega_{m,t}) = 1$ , with its cumulative distribution function and probability distribution function (c.d.f. and p.d.f. hereafter) denoted as  $F_t^m$  and  $f_t^m$  respectively. The risk factor, represented by the standard deviation of the corresponding normal distribution,  $\sigma_{m,t}$ , is assumed to be time varying and subject to an exogenous shock, which is considered as a proxy for the mortgage default risk changing over time:

$$\log \sigma_{m,t} = (1 - \rho_{\sigma_m}) \log \bar{\sigma}_m + \rho_{\sigma_m} \log \sigma_{m,t-1} + \epsilon_{\sigma_{m,t}}, \quad (4)$$

where  $\epsilon_{\sigma_{m,t}}$  is an i.i.d. process with standard deviation  $\sigma_{\sigma_m}$ . This setup is used commonly in the related literature to introduce mortgage default, as a fraction of the households will choose to stop paying their debt and select foreclosure on their mortgage when they are 'underwater.'<sup>22</sup>

Each period, the impatient household  $i$  will have an endogenously chosen threshold

as in [Garriga et al. \(2017\)](#) or [Greenwald \(2018\)](#).

<sup>19</sup>Refinancing is the most important factor determining prepayment: a cheaper mortgage rate available in the market than the mortgage rate currently makes refinancing attractive. Other reasons for prepayment include divorce and relocation; see [Meis \(2015\)](#). For simplicity, I abstract from other minor factors and use prepayment and refinancing interchangeably in this paper.

<sup>20</sup>Default can be considered as a specific form of prepayment in which the outstanding debt is returned via property sale; see [Meis \(2015\)](#). This description is also in line with the FHFA prepayment monitoring report, which states the action of the GSEs (Fannie Mae and Freddie Mac) when default happens has the same effect on MBS investors as a full prepayment. Below I define an adjusted prepayment rate to include the default situation and use it as the observable corresponding to the data.

<sup>21</sup>As also mentioned later, the insurance over the idiosyncratic shocks guarantees that the households own the same amount of housing stock at the end of each period, thus  $h_{t-1}^i = h_{t-1}$  at the beginning of period  $t$ .

<sup>22</sup>The interpretation of this idiosyncratic shock can be individual house price variation, for example in [Forlati and Lambertini \(2011\)](#), or different maintenance costs needed besides the common part as in [Elenev \(2017\)](#), among others.

policy  $\bar{\omega}_{m,t}^i$  just before the realization of her idiosyncratic quality shock, such that she will choose to repay when  $\omega_{m,t}^i \geq \bar{\omega}_{m,t}^i$  and default when  $0 \leq \omega_{m,t}^i < \bar{\omega}_{m,t}^i$ . In the latter case, the household does not repay on her current mortgage payment, losing the remainder of this debt contract, and gets her house taken away by the lender (financial intermediary). As shown later in the optimality condition, the household will compare the cost of honoring debt and losing the devalued housing stock and choose the threshold policy.

This idiosyncratic quality shock brings heterogeneity in households' wealth and therefore decisions. For tractability reasons, I follow the literature and assume that impatient households trade Arrow-Drew securities over this quality shock and thus are ensured against each other over it. Given this insurance available, all borrower households ex-ante choose the same threshold policy, therefore  $\bar{\omega}_{m,t}^i = \bar{\omega}_{m,t}$ . For notation, I denote  $\Delta_t = F_t^m(\bar{\omega}_{m,t})$  as the default rate of the borrower household in period  $t$ . I also denote the following function

$$G_{m,t} = G_{m,t}(\bar{\omega}_{m,t}) = \int_0^{\bar{\omega}_{m,t}} x dF_t^m(x),$$

so that  $G_{m,t}q_t^h h_{t-1}$  is the expected value of housing taken away by the financial intermediary under default, and  $(1 - G_{m,t})q_t^h h_{t-1}$  is the housing stock left in the hand of borrower households in expectation.<sup>23</sup>

Lastly, notice that impatient households who default on their debt are excluded from the mortgage market immediately under foreclosure, resulting in the population becoming less in the mortgage market after default. To keep the population constant, I make  $\bar{\Delta}_t = \Delta_t$  fraction of the borrower households who lost their housing stock under default at  $t$  enter the mortgage market again at the end of each period, and each finance the same amount of debt as the refinancing households defined later. Note that this entrance is exogenous and not taken into consideration when households choose the default threshold. Finally, at the end of each period, the housing stock value is evenly distributed across all impatient households according to the insurance term, and the borrower household  $i$  takes  $h_{i,t} = h_t$  units of houses to the next period.

**Prepayment.** For the non-default case after the quality shocks hit, the borrowers make mortgage payments to the bank, which contain two parts (in nominal terms): the promised interest from last period  $r_{t-1}m_{t-1}$  and the principal installment  $\varphi m_{t-1}$ , where  $r_{t-1}$  is the net mortgage rate in nominal terms, and  $\varphi \in (0, 1)$  denotes the fraction of the mortgage maturing each period. The coupon rate received at  $t$  can then be denoted as  $r_{m,t-1} = r_{t-1} + \varphi$ .

The mortgage debt is modeled as prepayable. This feature gives the fundamental reason for the household to adjust the mortgage stock, as well as following the institutional

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<sup>23</sup>The term  $G_{m,t}q_t^h h_{t-1}$  can also be interpreted as the total value of housing taken away from default households by the financial intermediary. This aggregation result can hold because of the representative agent setup given available insurance among impatient households, as well as the unity population weight within the cohort. The same argument is true for  $(1 - G_{m,t})q_t^h h_{t-1}$ .

design in the U.S. More specifically, borrower  $i$  can choose to prepay the balance of the mortgage loan in any period and end the existing contract. Then she also chooses a new loan size  $m_{i,t}^*$  with a net nominal rate  $r_t^*$ , subject to her collateral constraint

$$m_{i,t}^* \leq \theta_t^{LTV^*} \cdot q_t^h h_{i,t}^*$$

and the newly committed coupon to be paid next period is  $r_{m_t}^* = r_t^* + \varphi$ . The variable  $\theta_t^{LTV^*}$  is the contemporaneous loan-to-value ratio requirement applied to all new mortgage borrowers and is determined by

$$\theta_t^{LTV^*} = \bar{\theta}^{LTV^*} \cdot \exp(\epsilon_{\theta,t}) \quad (5)$$

where the exogenous i.i.d. shock process  $\epsilon_{\theta,t}$  has standard deviation  $\sigma_{\theta}$ .<sup>24</sup> In addition, her prepayment activity incurs a transaction cost  $\kappa_{i,t}^p m_{i,t}^*$ , where  $\kappa_{i,t}^p$  is drawn independently from the same distribution with its c.d.f. denoted as  $\Gamma_{\kappa^p,t}$  for all borrower households.<sup>25</sup> This setup allows borrowers to endogenously choose whether to prepay each period, and a fraction  $q_t \in (0, 1)$  of them will refinance.

The transaction cost distribution is depicted by the following Bernoulli-logistic mixture, such that  $\kappa_{i,t}^p$  is drawn from a logistic distribution parameterized by  $\mu_{\kappa}$  and  $s_{\kappa}$  with a probability of 1/4, and  $\kappa_{i,t}^p = \infty$  with a probability of 3/4, and the impatient households never repay facing this infinite prepayment cost. This follows [Greenwald \(2018\)](#), based on the observation that the quarterly prepayment rate varies within a maximum of about 20% over time given the dramatic change in the housing market and interest rate. The c.d.f of this mixture distribution is

$$\Gamma_{\kappa^p,t}(\kappa^p) = \frac{1}{4} \cdot \frac{1}{1 + \exp\left(-\frac{\kappa^p - \mu_{\kappa}}{s_{\kappa,t}}\right)}$$

with

$$s_{\kappa,t} = \bar{s}_{\kappa} \cdot \exp(\epsilon_{\kappa,t}), \quad (6)$$

and the i.i.d. process  $\epsilon_{\kappa,t}$  with standard deviation  $\sigma_{\kappa}$  capturing the exogenous change in the prepayment cost dispersion. Following [Greenwald \(2018\)](#), I also assume that borrowers pre-commit to a threshold cost level  $\bar{\kappa}_{i,t}^p$  influenced by the aggregate state but not the cross-section individual loan level. This assumption is to obtain tractability and aggregation of the model, and the probability of prepayment before the realization of  $\kappa_{i,t}^p$  across borrowers are identical given the financial market assumption of the model. In other words,  $\bar{\kappa}_{i,t}^p = \bar{\kappa}_t^p$ , and the unified prepayment rate is  $q_t = \Gamma_{\kappa^p,t}(\bar{\kappa}_t^p)$ .

<sup>24</sup>The contemporaneous loan-to-value ratio requirement is treated as an institutional factor exogenously determined, as in similar models such as [Iacoviello \(2005\)](#). This can be interpreted as exogenous changes in policies and regulations.

<sup>25</sup>This cost is introduced by assumption. It is born by the refinancing households and captures related costs such as inconvenience or possible high interest rates needed to be paid under prepayment.

**Representative Agent's Problem.** For convenience, define the total promised nominal payment of mortgage debt from last period as  $x_{m,t-1} = r_{m,t-1}m_{t-1}$ . Given the model setup, the laws of motions for mortgage related state variables are

$$m_t = \tilde{q}_t m_t^* + (1 - \tilde{q}_t)(1 - \varphi)\pi_t^{-1}m_{t-1} \quad (7)$$

$$x_{m,t} = \tilde{q}_t r_{m,t}^* m_t^* + (1 - \tilde{q}_t)(1 - \varphi)\pi_t^{-1}x_{m,t-1} \quad (8)$$

$$h_t = \tilde{q}_t h_t^* + (1 - \tilde{q}_t)(1 - G_{m,t})h_{t-1} \quad (9)$$

where I let  $\tilde{q}_t = (1 - \Delta_t)q_t + \bar{\Delta}_t$  as the 'default-adjusted' prepayment rate.

Now we can formulate the representative borrower's optimization problem, who endogenously chooses consumption  $c_t$ , labor supply  $n_t$ , newly issued stock of mortgage  $m_t^*$  and new housing size  $h_t^*$ , default rate  $\Delta_t$ , and a prepayment rate  $q_t$  to maximize (1) subject to the budget constraint

$$\begin{aligned} \underbrace{c_t}_{\text{cons.}} \leq & \underbrace{(1 - \tau_y)w_t n_t}_{\text{labor income}} - \underbrace{(1 - \Delta_t)x_{m,t-1}\pi_t^{-1}}_{\text{coupon payment}} + \underbrace{\tau_y(1 - \Delta_t)(x_{m,t-1} - \varphi)\pi_t^{-1}}_{\text{tax deduction}} \\ & + \underbrace{(1 - \Delta_t)q_t(m_t^* - (1 - \varphi)\pi_t^{-1}m_{t-1}) + \bar{\Delta}_t m_t^*}_{\text{new debt issuance}} - \underbrace{\delta_h q_t^h h_{t-1}}_{\text{maintenance}} \\ & - \underbrace{q_t q_t^h ((1 - \Delta_t)h_t^* - (1 - G_{m,t})h_{t-1}) - \bar{\Delta}_t q_t^h h_t^*}_{\text{housing adjustment}} - \underbrace{(\Psi(q_t) - \bar{\Psi}_t)m_t^*}_{\text{refinancing cost}} + \underbrace{T_t}_{\text{trans. payment}} \end{aligned} \quad (10)$$

and the collateral constraint in the representative agent's expression

$$m_t^* \leq \theta_t^{LTV^*} \cdot q_t^h h_t^* \quad (11)$$

as well as the state variables' laws of motion (7), (8), and (9). The function  $\Psi$  denotes the transaction cost paid under refinancing following earlier assumptions on this cost, which is the average of all possible proportional costs less than the cut-off value  $\bar{\kappa}^p$  chosen by the household, i.e.

$$\Psi(q_t) = \int_0^{\bar{\kappa}^p} x d\Gamma_{\kappa^p}(x) = \mathbb{E}(\kappa^p | \kappa^p < \bar{\kappa}^p).$$

The term  $\bar{\Psi}_t$  rebates the refinancing cost to the household, reflecting a non-pecuniary nature of this cost, as documented by [Greenwald \(2018\)](#).  $T_t$  rebates the taxes paid by the household in a lump-sum fashion.<sup>26</sup>

## Patient Households

The representative patient household's problem is standard with several assumptions. First its housing stock is held fixed over time. This assumption implies that the marginal buyers in the housing market are always from the borrower household cohort, and is

<sup>26</sup>Technically, these terms make the model steady state more convenient to solve while still letting agents internalize the tax incentive and refinancing cost at the margin.

used in the literature to prevent large unrealistic housing flows between the two types of households, as discussed in [Piazzesi and Schneider \(2016\)](#), for instance. Second, I assume that savers own their houses, and always save while the impatient households never save. They do need to pay housing maintenance costs like the borrowers to keep their houses at the beginning of each period.

Also, there is a one-period risk-less bond  $b'_t$  traded among saver households, which has a quantity zero in equilibrium given the representative agent setup. It is used to fulfill the short-term interest rate policy by the monetary authority. The corresponding gross nominal rate is denoted as  $R_t^{ib}$  ('inter-bank'). Denote the net nominal rate on deposits as  $r'_t$ . No arbitrage implies  $R_t^{ib} = 1 + r'_t$ .

Thus, a representative saver household chooses consumption  $c'_t$ , hours worked  $n'_t$ , the deposit  $d'_t$  to maximize the expected lifetime utility function

$$\mathbb{E}_t \sum_{k=0}^{\infty} \beta'^k z_{t+k} \left\{ \log(c'_{t+k} - \eta c'_{t+k-1}) + \bar{j} \log(\bar{h}') - \nu' \frac{n'^{1+\chi'}_{t+k}}{1+\chi'} \right\} \quad (12)$$

subject to the household's budget constraint (in each period  $t$ )

$$\begin{aligned} c'_t \leq & (1 - \tau_y) w'_t n'_t + (1 + r'_{t-1}) \pi_t^{-1} d'_{t-1} - d_t \\ & + (R_{t-1}^{ib} \pi_t^{-1} b'_{t-1} - b'_t) - \delta_h q_t^h h'_{t-1} - (h'_t - h'_{t-1}) + \Pi'_t + T'_t \end{aligned} \quad (13)$$

where  $\Pi'_t$  and  $T'_t$  are profits from intermediate firms owned by the patient households and the lump-sum transfer rebating taxes. Also,  $h'_t = \bar{h}'$  by the assumption stated above.

Before finishing the description of the household block, it is useful to define the stochastic discount factors for these two agents as  $\Lambda_{t,t+j} = \beta^j \frac{\lambda_{t+j}}{\lambda_t}$  and  $\Lambda'_{t,t+j} = \beta'^j \frac{\lambda'_{t+j}}{\lambda'_t}$  with  $j \geq 0$ , where  $\lambda_t, \lambda'_t$  are the Lagrangian multipliers on the impatient and patient households' budget constraints and equal to their marginal utilities of consumption respectively.

## 2.2 Entrepreneurs

The entrepreneur cohort is modeled according to the canonical work by BGG, with an important difference in the participation constraint of the lender (financial intermediary) stated later. Entrepreneurs take address on the unit interval, which facilitates aggregation, and the population weight for this cohort of agents is  $\gamma_e$ . All entrepreneurs are ex-ante identical. In period  $t$ , each of them is endowed with an investment project: she obtains capital stock  $k_t(i)$  from the capital producer at a unit price  $q_t^k$ , rents it to the intermediate goods producer; in the next period  $t+1$  after the aggregate shocks realize in the economy, she gets the rental return of capital  $r_{t+1}^k$  from the intermediate goods producer, and sells the depreciated capital stock  $(1 - \delta_k)k_t(i)$  to the capital producer for price  $q_{t+1}^k$ .

Because the net worth an entrepreneur holds is smaller than the size of the project, each of them needs external funds to finance it. More specifically, the entrepreneur indexed by  $i$  borrows  $b_t(i)$  from the financial intermediary. It uses its own net worth  $NW_t(i)$  and the

external firm loan  $b_t(i)$  to purchase productive capital  $k_t(i)$  and rent it to the intermediate producer. The total value of capital purchased by entrepreneur  $i$  is  $q_t^k k_t(i) = NW_t(i) + b_t(i)$ . In aggregate terms,<sup>27</sup> this can be written as

$$q_t^k k_t = NW_t + b_t. \quad (14)$$

The entrepreneurs also choose the utilization rate of physical capital  $u_t$  with a cost  $\Psi_k(u_t)k_t$ . This determines the effective capital stock  $k_{u,t}$  rented to the intermediate good producer, where  $k_{u,t} = u_t k_t$ . The utilization rate is unity in the deterministic steady state, and utilization cost function has the following property, such that  $\Psi_k(1) = 0$  and  $\frac{\Psi_k''(1)}{\Psi_k'(1)} = \frac{\psi}{1-\psi}$ , with  $\psi \in [0, 1)$ .<sup>28</sup> As  $\psi$  approaches unity, the elasticity of the utilization rate cost increases to infinity, which corresponds to the situation with the utilization margin shut down. The corresponding optimization problem choosing the utilization rate is

$$\max_{u_t} r_t^k k_{u,t} - \Psi_k(u_t)k_{t-1},$$

and the real aggregate return on an entrepreneur's investment project at  $t + 1$  is

$$\tilde{R}_{t+1}^k = \frac{u_t r_{t+1}^k + (1 - \delta_k) q_{t+1}^k}{q_t^k}. \quad (15)$$

After receiving the aggregate return  $\tilde{R}_t^k$  on the project at time  $t$ , entrepreneur  $i$  receives an idiosyncratic shock  $\omega_t$  (with its realization as  $\omega_t^i$ ), and the ex-post return of entrepreneur  $i$  becomes  $\omega_t^i \tilde{R}_t^k$ . The shock  $\omega_t$  is drawn from a time-varying distribution with non-negative support and  $\mathbb{E}_{t-1}(\omega_t) = 1$ . This distribution is identical for all entrepreneurs. Let  $f_t$  and  $F_t$  denote its p.d.f. and c.d.f. respectively. The standard deviation of  $\omega_t$  is denoted as  $\sigma_{e,t}$  which is the source of the distribution time-variation. It represents the investment project's riskiness and evolves according to the following law of motion:

$$\log \sigma_{e,t} = (1 - \rho_{\sigma_e}) \log \bar{\sigma}_e + \rho_{\sigma_e} \log \sigma_{e,t-1} + \epsilon_{\sigma_{e,t}} \quad (16)$$

with  $\epsilon_{\sigma_{e,t}}$  being i.i.d. with variance  $\sigma_{\sigma_e}^2$ .

The contract between entrepreneur  $i$  and the financial intermediary follows the costly state verification framework. At  $t - 1$ , the two parties sign the following loan contract

$$b_{t-1}(i) R_{t-1}^Z(i) = \bar{\omega}_t(i) \tilde{R}_t^k q_{t-1}^k k_{t-1}(i)$$

where the endogenous default threshold  $\bar{\omega}_t(i)$  is period  $t$  state-contingent as in the work by BGG, but contractual interest rate  $R_{t-1}^Z(i)$  is predetermined. The capital return rate in

<sup>27</sup>Note that the aggregate variables are also equivalent to the representative entrepreneur's related variables, given that the population weight of entrepreneurs is unity within the cohort.

<sup>28</sup>The utilization cost specification is standard in the literature following for instance [Leeper et al. \(2017\)](#), [Smets and Wouters \(2007\)](#), and [Christiano et al. \(2005\)](#).

the contract is

$$\tilde{R}_t^k = \tilde{R}_t^k / \exp(\epsilon_{rp,t}) \quad (17)$$

where the disturbance  $\epsilon_{rp,t}$  serves as an exogenous premium shock with standard deviation  $\sigma_{rp}$ . The entrepreneurs make bankruptcy decisions after observing their own idiosyncratic shock realizations. More specifically if  $0 < \omega_t^i < \bar{\omega}_t(i)$ , the entrepreneur  $i$  defaults and the return from her project is collected by the lender with a proportional monitoring cost  $\mu$ ; if  $\omega_t^i \geq \bar{\omega}_t(i)$ , she repays the loan according to the contract above. Entrepreneurs are perfectly ensured against each other over bankruptcy, so they make the same optimizing decision prior to the idiosyncratic shock.<sup>29</sup> Therefore, they will choose the same default threshold menu and contractual interest rate in optimality ex-ante, i.e.  $\bar{\omega}_t(i) = \bar{\omega}_t$  for given aggregate state at  $t$ , and  $R_{t-1}^Z(i) = R_{t-1}^Z$ . For notation convenience, denote the following function

$$G_t(\bar{\omega}_t) \equiv \int_0^{\bar{\omega}_t} x f_t(x) dx$$

where  $G_t(\bar{\omega}_t)$  can be interpreted as the expected gross return under default.

Lastly, entrepreneurs survive to the next period with an exogenous constant probability  $\gamma \in (0, 1)$  after the default stage and bankruptcy insurance payoff.<sup>30</sup> In case of surviving, the entrepreneurs consume nothing and reinvest their net worth in the following period. The exiting entrepreneurs, on the other hand, consume all the equity they own and become patient households. To keep the entrepreneur population constant (also the households), a fraction  $1 - \gamma$  of new entrepreneurs from the patient households will replace those exiting and each of them has an initial endowment  $W_t^e$  transferred from the patient household. These new entrants pool their endowment with the existing entrepreneurs, and the insurance contract among all the agents in the entrepreneur cohort realizes, making sure that they will have the same net worth level before entering the next period. Let  $V_t$  be the aggregate equity held by entrepreneurs before the exit shock realizes, which is

$$V_t = \left[ \int_{\bar{\omega}_t}^{\infty} (x - \bar{\omega}_t) f_t(x) dx \right] R_t^k q_{t-1}^k k_{t-1}. \quad (18)$$

The aggregate net worth at the end of period  $t$  used to finance projects in the next period is

$$NW_t = \gamma V_t + (1 - \gamma) W_t^e. \quad (19)$$

Also, it is convenient to define the following function as in the original BGG paper:

$$\Gamma_t(\bar{\omega}_t) \equiv \bar{\omega}_t \int_{\bar{\omega}_t}^{\infty} f_t(x) dx + \int_0^{\bar{\omega}_t} x f_t(x) dx = \bar{\omega}_t [1 - F_t(\bar{\omega}_t)] + G_t(\bar{\omega}_t)$$

<sup>29</sup>I introduce this insurance market to make the framework tractable, following the common practice in the literature.

<sup>30</sup>This setup is to eliminate the situation that the entrepreneur accumulates enough net worth to be self-financed.



where  $\Gamma_t(\bar{\omega}_t)$  can be interpreted as the expected gross share of profit going to the lender before taking the monitoring cost into account.<sup>31</sup> Also, denote  $\kappa_t(i) = \frac{q_t^k k_t(i)}{NW_t(i)}$  as the entrepreneur  $i$ 's leverage. In a symmetric equilibrium it follows that  $\kappa_t(i) = \kappa_t$ .<sup>32</sup>

## 2.3 Financial Intermediary

There is one financial intermediary (FI hereafter) in the model economy,<sup>33</sup> and it is assumed to be owned by the saver households. In each period, it receives deposits, with the aggregate stock being  $D'_t = \gamma_s \int_0^1 d'_t di$  at  $t$ , from the saver households, which is the only source of its funding. The FI issues long-term debt ("mortgages") to impatient households, with the aggregate debt stock denoted as  $M_t = \gamma_h \int_0^1 m_t di$ , and one-period business loans to entrepreneurs, with the aggregate quantity denoted as  $B_t = \gamma_e \int_0^1 b_t di$ . These variables are all in real terms.

### Pooling and Participation

The FI pools the mortgage loans together in order to eliminate the idiosyncratic default risk, and an ex-ante expected return rate at  $t$  is determined as

$$R_{m,t}^{I,a} = \mathbb{E}_t \left\{ (1 - \Delta_{t+1})(R_t - \varrho_{t+1}r_t) + (1 - \mu_m) \frac{Q_{t+1}^h h_t}{m_t} G_{m,t+1} \right\}, \quad (20)$$

where  $R_{m,t}^{I,a}$  is the ex-ante expected nominal return rate of mortgage loans, and  $R_t = 1 + r_t$  is the gross nominal mortgage interest rate on the existing mortgage stock. The parameter  $\mu_m$  represents the cost incurred when the lender recovers the collateral under default, and the impatient households who default on their mortgage loans bear this cost defined as

$$cost_{m,t} = \mu_m G_{m,t} q_t^h h_{t-1}.$$

Equation (20) is a participation constraint for the financial intermediary to lend in the mortgage market. Here I make the assumption that the lender (FI) can only write a contract in expectation. The ex-post return of mortgage loan  $R_{m,t+1}^I$  (corresponding to  $R_{m,t}^{I,a}$ ) is determined by the realized shocks in the current period, and the following relationship holds:

$$R_{m,t}^{I,a} = \mathbb{E}_t R_{m,t+1}^I. \quad (21)$$

<sup>31</sup>Later the expressions  $F_t$ ,  $G_t$  and  $\Gamma_t$  are also used in place of  $F_t(\bar{\omega}_t)$ ,  $G_t(\bar{\omega}_t)$  and  $\Gamma_t(\bar{\omega}_t)$  in order to save notation.

<sup>32</sup>I suppress details of the representative entrepreneur's optimization problem to the appendix. Briefly, the entrepreneur chooses  $\bar{\omega}_{t+1}$  and  $\kappa_t$  to maximize (18) subject to the FI's participation constraint (23).

<sup>33</sup>This financial intermediary represents the collection of all financial institutions in reality. The consolidation of the financial system to one sector in the model follows [Iacoviello \(2015\)](#), for instance.

<sup>34</sup>It follows that  $R_{m,t+1}^I = (1 - \Delta_{t+1})R_t + (1 - \mu_m) \frac{Q_{t+1}^h h_t}{m_t} G_{m,t+1}$ .

The lender will take the loss (or gain) at period  $t + 1$  as

$$loss_{m,t+1} = \varrho_m loss_{m,t} + (R_{m,t+1}^I - R_{m,t}^{I,a}) \pi_{t+1}^{-1} M_t \quad (22)$$

where  $\varrho_{lm} \in [0, 1)$  shows the persistence of the loss. I introduce the lagged term in (22) by assuming that the contemporaneous loss can possibly be carried to future periods. This term will affect the net worth accumulation of FI overtime.<sup>35</sup>

A similar setup applies for business loans. Specifically, in order to make the financial intermediary participate in lending to entrepreneurs, the following participation constraint has to hold at  $t - 1$ :

$$R_{e,t}^{I,a} = \mathbb{E}_t \left\{ R_t^Z (1 - F_{t+1}) + (1 - \mu) G_{t+1} \tilde{R}_{t+1}^k q_t^k k_t / b_t \right\} \cdot \mathbb{E}_t \pi_{t+1} \quad (23)$$

where  $R_{e,t}^{I,a}$  is the ex-ante business loan rate in gross and nominal terms. It is important to point out that the assumption again only allows the lender to write the contract in expectation. The parameter  $\mu$  represents the monitoring cost incurred when the lender recovers the return on defaulted projects, and the entrepreneurs who default on their loans bear this cost defined as

$$cost_{e,t} = \mu G_t(\bar{\omega}_t) \tilde{R}_t^k q_{t-1}^k k_{t-1}.$$

After the realization of period  $t + 1$  shocks, the ex-post return of business loans  $R_{e,t+1}^I$  satisfies the following relationship:

$$R_{e,t}^{I,a} = \mathbb{E}_t R_{e,t+1}^I \quad (24)$$

and the lender will take the loss (or gain) as

$$loss_{e,t+1} = \varrho_{le} loss_{e,t} + (R_{e,t+1}^I - R_{e,t}^{I,a}) \pi_{t+1}^{-1} B_t \quad (25)$$

where  $\varrho_{le} \in [0, 1)$  shows the persistence of loss, and  $B_t$  is the aggregate business loan stock (in real terms) defined later in the financial intermediary section. I introduce this loss persistence by assumption to allow possible loss spillover to future periods. Same as the mortgage lending case, this term will also affect the lender's net worth accumulation overtime.

Assume for now that no fundamental shocks occurred in the economy. It follows that the expected and realized return rates are the same, i.e.  $R_{m,t+1}^I = R_{m,t}^{I,a}$  and  $R_{e,t+1}^I = R_{e,t}^{I,a}$  according to equations (20), (21), (23), and (24). I use anticipated rates in the following two cash flow expressions as they are the ones the FI takes into account when optimally choosing loan quantities supplied by assumption.

At period  $t + 1$ , the FI receives  $R_{m,t}^{I,a}$  as the nominal payment on each unit of the non-defaulted and non-prepaid long-term debt. The FI also issues new loans to the two types

<sup>35</sup>Potentially the lender can go bankrupt if the loss is larger than its net worth level. I assume the lender has an infinitely deep pocket so that it will not be an issue.

<sup>36</sup>It follows that  $R_{e,t+1}^I / \pi_{t+1} = R_t^Z (1 - F_{t+1}(\bar{\omega}_{t+1})) + (1 - \mu) G_{t+1}(\bar{\omega}_{t+1}) \tilde{R}_{t+1}^k q_t^k k_t / b_t$ .

of borrowers each period. In aggregate the mortgage loan cash flow in the FI at  $t + 1$  is

$$R_{m,t}^{I,a} \pi_{t+1}^{-1} M_t - M_{t+1}.$$

The entrepreneur's short-term gross nominal interest rate on debt is  $R_{e,t}^{I,a}$  and forms the other source of the FI's income. The business loan cash flow at  $t + 1$  is

$$R_{e,t}^{I,a} \pi_{t+1}^{-1} B_t - B_{t+1}.$$

On the deposit side, the cash flow can be written as below given the gross rate  $R'_t = 1 + r'_t$ :

$$-R'_t \pi_{t+1}^{-1} D'_t + D'_{t+1}.$$

When there are exogenous shocks in the economy, the anticipated and realized return rates are no longer the same. The FI still makes optimized decisions based on anticipated turns, while the realized returns created potential losses (or gains) to the FI according to equations (22) and (25). Although not affecting the FI's marginal decisions, these losses will show up in the accumulation of FI's net worth defined later in (28) and vary its balance sheet strength over time.

### FI's Portfolio Choice

To introduce loan portfolio choice for the FI, I follow [Bae \(2012\)](#) and let the bank choose the share of mortgage loans in its portfolio, denoted as  $s_t$ . Thus  $s_t \in (0, 1)$ , and business loans take  $1 - s_t$  fraction of the FI's asset. To allow for sluggish portfolio change, I introduce a quadratic adjustment cost when the FI changes the share  $s_t$ , which is external to the FI and takes the following form:

$$acc_{s,t} = \frac{\phi_s}{2} (s_t - s_{t-1})^2 S_t$$

where the variable  $S_t = M_t + B_t$  denotes the total assets held by the FI.<sup>37</sup> The unit costs of intermediating the household debt and entrepreneurial business loans,  $\Theta_t$  and  $\Xi_t$  respectively, take the following form

$$\Theta_t = \frac{1}{2} \Phi_{pm,t} s_t, \quad \Xi_t = \frac{1}{2} \Phi_{pe,t} (1 - s_t)$$

with

$$\Phi_{pm,t} = \bar{\Phi}_{pm} \left( \frac{\Delta_t}{\bar{\Delta}} \right)^{\theta_d} \left( 1 + \left( \frac{Q_t}{\bar{Q}} - 1 \right)^2 \right)^{\theta_p}, \quad \Phi_{pe,t} = \bar{\Phi}_{pe} \left( \frac{\Delta_{e,t}}{\bar{\Delta}_e} \right)^{\zeta_d}$$

<sup>37</sup>The adjustment cost captures the fact that loans are commitments and take time to adjust to shocks such as monetary policy changes, as discussed for instance in [Bernanke and Blinder \(1992\)](#), and more recently [Black and Rosen \(2016\)](#).

where  $\Delta_{e,t} = F(\bar{\omega}_t)$  is the entrepreneur default rate, and the barred variable dropping the time subscript denotes its value in the deterministic steady state. I make the deviation of the prepayment rate from its steady state matter for the intermediation cost change according to the statement in the FHFA prepayment monitoring report. The parameters  $\bar{\Phi}_{pm}, \bar{\Phi}_{pe}$  are calibrated to match the data premia (according to MBS and BAA spreads), and  $\theta_d, \theta_p, \zeta_d$  are all non-negative.

As seen above, the loan supply cost functions are assumed to be related to fundamental changes in the corresponding sectors. The mortgage debt intermediation cost is positively related to the corresponding loan default rate and prepayment rate deviations, while the business sector debt supply cost moves positively with the entrepreneur default rate. The parameters on the powers can be interpreted as the supply cost elasticities with respect to corresponding risks. This modeling captures the fact that the intermediation costs are influenced by the relevant risks of the underlying asset. For instance, the mortgage securitization cost relates to the prepayment rate variation.

Each period, the FI uses the retained earnings to accumulate its net worth, or its equity, denoted as  $K_t^b$ . Following the literature, I let the net worth ‘depreciate’ at a rate of  $\delta_{kb}$  each period. This part of the resource transfers to the patient households.<sup>38</sup>

## FI’s Problem

Now we can formulate the FI’s optimization problem. For notation, a variable with tilde represents the real interest rate of the corresponding nominal rate, for instance  $\tilde{R}_{m,t}^{I,a} = R_{m,t}^{I,a} / \mathbb{E}_t \pi_{t+1}$ . The FI’s objective is to choose the amount of loans and deposits, as well as the shares of the two assets in its balance sheet, in order to maximize the discounted sum of real cash flows:

$$\max_{M_t, B_t, D'_t, s_t} \mathbb{E}_0 \sum_{t=0}^{\infty} \Lambda'_{0,t+1} \left[ \tilde{R}_{m,t}^{I,a} M_t - M_{t+1} + \tilde{R}_{e,t}^{I,a} B_t - B_{t+1} - \tilde{R}'_t D'_t + D'_{t+1} - \Theta_t M_t - \Xi_t B_t \right. \\ \left. + K_{t+1}^b - K_t^b - \frac{\phi_b}{2} \left( \frac{K_t^b}{S_t} - v_b \right)^2 K_t^b - acc_{s,t} \right]$$

subject to the portfolio choice constraints<sup>39</sup>

$$M_t \leq s_t (D'_t + K_t^b) \quad (26)$$

$$B_t \leq (1 - s_t) (D'_t + K_t^b) \quad (27)$$

<sup>38</sup>Technically, this setup keeps the model stationary by not letting the FI’s net worth accumulate to infinity.

<sup>39</sup>The linear combination of the two portfolio constraints amounts to a standard balance sheet constraint:  $M_t + B_t \leq D'_t + K_t^b$ .

where  $K_t^b$  is the FI's net worth (in real term) accumulated according to the following equation:

$$K_t^b = (1 - \delta_{kb})K_{t-1}^b + \tilde{R}_{m,t-1}^{I,a}M_{t-1} + \tilde{R}_{e,t-1}^{I,a}B_{t-1} - \tilde{R}'_{t-1}D'_{t-1} - K_{t-1}^b - \Theta_{t-1}M_{t-1} - \Xi_{t-1}B_{t-1} - \frac{\phi_b}{2} \left( \frac{K_t^b}{S_t} - \nu_b \right)^2 K_t^b - acc_{s,t} - \gamma_h loss_{m,t} - \gamma_e loss_{e,t} \quad (28)$$

with the parameter  $\nu_b \in (0, 1)$  representing the capital requirement ratio. The term

$$acc_{b,t} = \frac{\phi_b}{2} \left( \frac{K_t^b}{S_t} - \nu_b \right)^2 K_t^b$$

in the FI's objective function is the adjustment cost incurred by deviating from the capital requirement regulation and parameter  $\phi_b \geq 0$  governs the cost of FI adjusting its net worth to asset ratio away from the regulation target. Lastly, for later use, denote the real resource used up in the intermediation process as

$$cost_t^{FI} = \Theta_t M_t + \Xi_t B_t. \quad (29)$$

## 2.4 Production Sector

Three kinds of firms exist in the production sector: an intermediate goods producer, a continuum of retailers with unity measure, and a capital producer. Without loss of generality, they are assumed to be owned by the patient households.

The intermediate good  $Y_t$  is produced according to a Cobb-Douglas technology:

$$Y_t = A_t \left( N_t'^\alpha N_t^{1-\alpha} \right)^{1-\mu_c} K_{u,t}^{\mu_c} \quad (30)$$

where  $\alpha \in (0, 1)$  denotes the share of patient households' labor needed in the labor force, and  $\mu_c \in (0, 1)$  is the capital contribution during intermediate good production. Also,  $K_{u,t} = \gamma_e k_{u,t}$ . The total factor productivity follows the law of motion:

$$\log A_t = \rho_a \log A_{t-1} + (1 - \rho_a) \log \bar{A} + \epsilon_{a,t} \quad (31)$$

with  $\epsilon_{a,t}$  as the exogenous shock driving the changes in  $A_t$ , and its standard deviation is  $\sigma_a$ .

Retailers are subject to monopolistic competition, and differentiate the intermediate good to the final retail good at no cost. Retailers' price-resetting problem is *à la Calvo*, with the price resetting probability each period being  $1 - \theta$  and  $\theta \in [0, 1)$ .<sup>40</sup> The details of the retailer's problem are standard and suppressed to the appendix. The corresponding

<sup>40</sup>This modeling follows canonical work by Calvo (1983) and Yun (1996). The detailed specification of retailers follows the New Keynesian literature such as Bernanke et al. (1999) and Smets and Wouters (2007).

log-linearized Phillips curve is

$$\hat{\pi}_t - \iota \hat{\pi}_{t-1} = -\kappa \hat{X}_t + \beta' (\mathbb{E}_t \hat{\pi}_{t+1} - \iota \hat{\pi}_t) + \epsilon_{p,t} \quad (32)$$

with  $X_t = \frac{P_t^w}{P_t}$  as the average mark-up of the final good price index  $P_t$  over the whole sale price  $P_t^w$ , and  $\kappa = \frac{(1-\theta)(1-\beta'\theta)}{\theta} > 0$  is the slope of the linearized Phillips curve. Parameter  $\iota \in [0, 1]$  denotes the degree of price indexation. The variables with hats denote the percentage deviation from their deterministic steady state values. The shock  $\epsilon_{p,t}$  denotes the mark-up shock and follows a normal distribution with mean zero and standard deviation  $\sigma_p$ .

The capital producer purchases the depreciated capital stock from the entrepreneur and refurbishes it to new capital stock, which will be sold back to the entrepreneurs for next period's production.<sup>41</sup> More specifically, the capital producer uses  $\left(1 + \Phi_k \left(\frac{I_t}{I_{t-1}}\right)\right) I_t$  units of the consumption good and produces  $I_t^*$  units of the capital good, such that  $I_t^* = \exp(a_t^i) I_t$ , where the exogenous disturbance  $a_t^i$  is the investment specific shock affecting the efficiency of capital accumulation, following the AR(1) process

$$a_t^i = \rho_i a_{t-1}^i + \epsilon_{i,t}.$$

where the i.i.d. process  $\epsilon_{i,t}$  has standard deviation  $\sigma_i$ . An adjustment cost is incurred in the production as  $\Phi_k \left(\frac{I_t}{I_{t-1}}\right) I_t$ , and the functional form of  $\Phi_k$  is defined as<sup>42</sup>

$$\Phi_k(x) = \frac{\phi_k}{2} (x - 1)^2$$

with  $\phi_k \geq 0$ . The profit maximization problem for the capital producer is then

$$\max_{\{I_t\}} \mathbb{E}_0 \sum_{t=0}^{\infty} \Lambda'_{0,t} \left\{ q_t^k I_t^* - \left(1 + \Phi_k \left(\frac{I_t}{I_{t-1}}\right)\right) I_t \right\}.$$

Denote the capital depreciation rate as  $\delta_k \in (0, 1]$ . The law of motion for the aggregate capital stock accumulation is

$$K_t = (1 - \delta_k) K_{t-1} + I_t^*. \quad (33)$$

<sup>41</sup>The capital producer setup is standard: see relevant parts in [Carlstrom et al. \(2016\)](#) for instance.

<sup>42</sup>In general, an adjustment cost function under this setup satisfies the following properties:

$$\Phi_k(1) = \Phi_k'(1) = 0; \quad \Phi_k''(1) = \phi_k \geq 0.$$

The quadratic adjustment cost function can satisfy these conditions.

## 2.5 Closing the Model

The following market clearing conditions hold in equilibrium. In the consumption good market.<sup>43</sup>

$$\tilde{Y}_t + \zeta cost_t + cost_t^{FI} + acc_{b,t} + acc_{s,t} = Y_t \quad (34)$$

where the resources used for consumption and capital accumulation are

$$\tilde{Y}_t = \gamma_h c_t + \gamma_s c'_t \delta_h q_t \bar{h} + I_t \left( 1 + \Phi_k \left( \frac{I_t}{I_{t-1}} \right) \right) + \Psi_k(u_t) K_t. \quad (35)$$

The housing market is assumed to have a fixed amount of housing stock. Thus:

$$\gamma_h h_t + \gamma_s \bar{h}' = \bar{H}. \quad (36)$$

Credit market clearing conditions are stated in the financial intermediary's optimizing problem above, as in (26) and (27), as well as the aggregation relationships for the FI's assets and liability.

Factors of production markets clear as

$$N_t = \gamma_h n_t, \quad N'_t = \gamma_s n'_t, \quad K_t = \gamma_e k_t. \quad (37)$$

Finally, the monetary authority conducts conventional interest rate policy defined by a Taylor rule

$$\hat{R}_t^{ib} = \rho_r \hat{R}_{t-1}^{ib} + (1 - \rho_r) [\rho_\pi \hat{\pi}_t + \rho_y (\hat{y}_t - \hat{y}_{t-1})] + a_t^{ib} \quad (38)$$

where  $a_t^{ib}$  is the interest rate shock which evolves according to the following process

$$a_t^{ib} = \rho_{ib} a_{t-1}^{ib} + \epsilon_{ib,t} \quad (39)$$

with the standard deviation of  $\epsilon_{ib,t}$  as  $\sigma_{ib}$  and  $\rho_{ib} \in [0, 1]$ .<sup>44</sup> The interbank rate, or the policy rate  $R_t^{ib}$  is defined according to the following non-arbitrage condition:

$$1 = \mathbb{E}_t \left\{ \Lambda'_{t,t+1} R_t^{ib} \pi_{t+1}^{-1} \right\}. \quad (40)$$

The monetary authority can also exert unconventional policy, which is modeled as a direct shock changing the rate on the newly issued mortgage loans, following [Pietrunti and Signoretti \(2017\)](#):

$$r_t^f = r_t^* + a_{f,t} \quad (41)$$

where  $r_t^f$  is the effective mortgage rate subject to the exogenous change in the policy

<sup>43</sup>The parameter  $\zeta \in (0, 1]$  represents the dead-weight loss incurred by default as a fraction of the output. Previous studies have chosen it to be 0.5 ([Ferrante, 2018](#)) or 1 ([Christiano et al., 2014](#); [Gomes et al., 2016](#)) or possibly other plausible values. I set it equal to 1 in the benchmark case, but the results are not sensitive to alternative values (result available upon request).

<sup>44</sup>The interest rate shock can be interpreted as the discretionary component of the monetary policy.

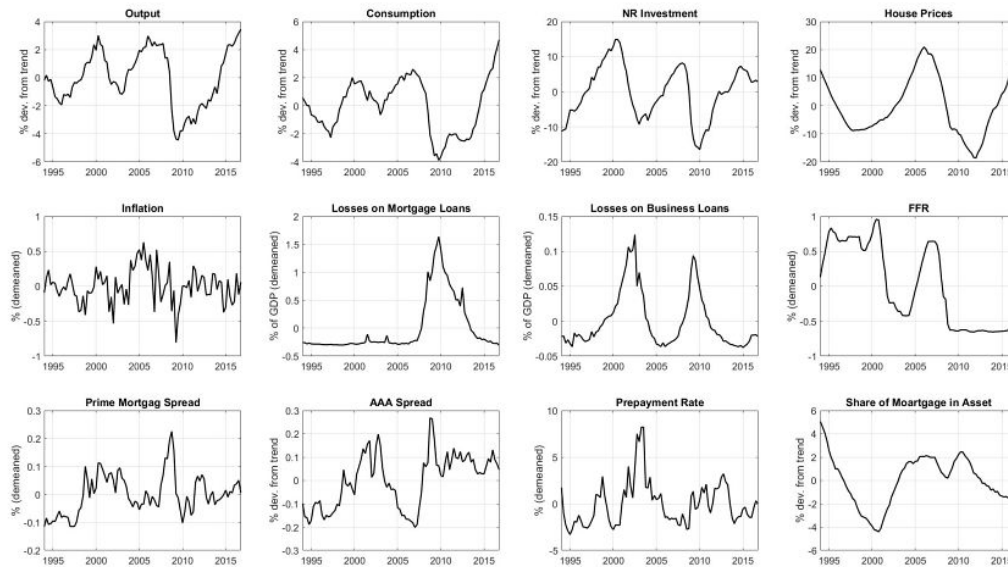
innovation. The variable  $a_{f,t}$  has mean zero and follows the AR(1) process as  $a_{f,t} = \rho_f a_{f,t-1} + \epsilon_{f,t}$ . This completes the description of the model.

### 3 Model Estimation

#### 3.1 Method and Data

To quantitatively evaluate the model, I calibrate some of the model parameters based on the data and relevant studies and estimate the other parameters of interest using Bayesian inference. The equilibrium conditions of the model are (log-)linearized around the deterministic steady state. As the solution of the model is rearranged into a linear state-space formulation, the Kalman filter is employed to obtain a likelihood function combined with the prior distributions chosen for the model parameters. This joint distribution is then sampled using the random-walk Metropolis algorithm to get the posterior distributions of the parameters. This estimation procedure is standard in the literature, as in [Iacoviello \(2015\)](#) and [Suh and Walker \(2016\)](#), for instance. I take 1,200,000 draws of the posterior distribution with the first 600,000 discarded, and run two parallel chains to check sampling convergence.

Figure 3: Data used in Estimation



I choose twelve observables to estimate the model.<sup>45</sup> They include five key aggregate variables (output, consumption, non-residential investment, house price, inflation rate), and seven others including mortgage and business loan losses over output, the federal

<sup>45</sup>I consider eleven structural shocks, and add an observation error to the mortgage share variable to avoid stochastic singularity in estimation. The related details are suppressed to appendix.



funds rate, the prime mortgage rate spread, the business loan liquidity spread, the prepayment rate, and the share of mortgage loans in the mortgage-business loan asset pool. The data are divided by the total population to convert to per capita terms if necessary. Also, the relevant data series are deflated using the GDP deflator. For data series with secular trends, I use their logarithms and then quadratically detrend each of them independently. The stationary variables are demeaned.<sup>46</sup> The sample period chosen to run the estimation is from 1994:Q1 to 2016:Q4.<sup>47</sup> The data series used in the estimation are plotted in Figure 3.

### 3.2 Calibrated Parameters

The discount factor for the patient household  $\beta'$  is set to 0.9925, implying a 3% real interest rate annually. The impatient household's discount factor is set to 0.965 as a conservative and standard value in the literature, guaranteeing a binding borrowing constraint near the steady state, and the model dynamics are generally unchanged with its alternative values in the reasonable neighborhood. Inverse Frisch labor elasticities  $\chi$  and  $\chi'$  are both set to 1. The housing service preference  $j$  is set to 0.0985,<sup>48</sup> so that in steady state the patient household's property value to income ratio is 11.4 quarterly in accordance with the 1998 Survey of Consumer Finances (SCF hereafter), as stated in [Greenwald \(2018\)](#). The parameter value is also close to the counterpart in [Iacoviello and Neri \(2010\)](#) (0.12). The population weight of the borrower is set to  $\gamma_h = 0.319$  following [Greenwald \(2018\)](#), defining the impatient households as those with a house and mortgage but with liquid asset holdings less than two months' income, according to the 1998 SCF. The patient households' population weight is  $\gamma_s = 1 - \gamma_h$  correspondingly.

For the housing sector, the depreciation rate of housing stock  $\delta_h$  is set to 0.005, implying an annual depreciation rate of 2%.<sup>49</sup> For the housing stock in the economy, I calibrate  $\bar{H} = 6.6$ , so that in steady state the real house price is unity. The contemporaneous loan-to-value ratio  $\bar{\theta}^{LTV*}$  is set to 0.84, according to the newly originated mortgage data for various percentiles documented by [Greenwald \(2018\)](#).

The mortgage default rate is set to 0.375%, implying an annualized delinquency rate of 1.5%, close to the mean of the single-family mortgage delinquency rate one or two years prior to the crisis. The standard deviation of the idiosyncratic mortgage default distribution  $\bar{\sigma}_m$  is set to 0.17 in steady state, with the direct cost fraction incurred by mortgage

<sup>46</sup>The source and more details of the data can be found in the appendix.

<sup>47</sup>The starting point of the sample is chosen based on the availability of the conditional prepayment rate data. It contains the zero lower bound period, however as noted by [Hirose and Inoue \(2016\)](#), the parameter estimates are in general unbiased albeit the monetary policy related parameters are slightly biased. An alternative approach is to use a shadow rate measure, for instance, by [Wu and Xia \(2016\)](#), and preliminary estimation results show minor difference with those in the main text.

<sup>48</sup>This parameter is set the same for both types of households. Although the patient households' housing stock is kept fixed, I make this fixed quantity in line with their optimality condition if they were able to adjust the housing stock.

<sup>49</sup>The housing stock depreciation rate is smaller than the one with physical capital as stated later. These two parameter values are standard in the literature based on the relevant data's first moments.

Table 1: Calibrated Parameters

Description	Parameter	Value	Target/Source
Demographic and Preference			
Discount factor - imp hhs	$\beta$	0.9925	3% annual real rate
Discount factor - pnt hhs	$\beta'$	0.965	Greenwald (2018), Iacoviello and Neri (2010)
Housing preference (both hhs)	$j$	0.0985	1998 SCF (Greenwald, 2018)
Frisch inverse labor elasticity-imp hhs	$\chi$	1	Standard
Frisch inverse labor elasticity-pnt hhs	$\chi'$	1	Standard
Population weight - imp hhs	$\gamma_h$	0.319	1998 SCF (Greenwald, 2018)
Population weight - pnt hhs	$\gamma_s$	0.681	$1 - \gamma_h$
Housing and Mortgage			
Depreciation rate - housing stock	$\delta_h$	0.005	2% annual rate
Housing stock - aggregate	$\bar{H}$	6.6	$q^h = 1$ in steady state
Current LTV ratio	$\theta^{LTV*}$	0.84	Greenwald (2018)
Default rate - mortgage loan	$\bar{\Delta}$	0.375%	Single-family residential mortgage delinquency rate 2005
Mortgage loan risk	$\bar{\sigma}_m$	0.17	Default premium, share of mortgage loans in asset
Mortgage default cost fraction	$\mu_m$	0.238	Mortgage loan default premium
Mortgage amortization rate	$\varphi$	0.435%	Greenwald (2018)
Prepayment cost distribution mean	$\mu_\kappa$	0.348	Greenwald (2018)
Prepayment cost distribution scale	$s_\kappa$	0.152	Greenwald (2018)
Entrepreneurs and Business Sector			
Population rate - entrepreneurs	$\gamma_e$	0.681	Same as patient households
Depreciation rate - capital stock	$\delta_k$	0.025	Standard
Survival rate of entrepreneurs	$\gamma$	0.94	16.7 quarters of business life span
Default rate - entrepreneur	$\bar{\Delta}_e$	0.0875%	3.5% annual rate
Business loan risk	$\bar{\sigma}_e$	1.25	Default premium, share of business loans in asset
Monitoring cost fraction	$\mu$	0.7	Business loan default premium
Financial Intermediary			
Capital requirement ratio	$\nu$	0.09	Gerali et al. (2010)
Intermediation cost - mortgage loan	$\bar{\Phi}_{pm}$	0.007	1% liquidity premium, mortgage loan
Intermediation cost - business loan	$\bar{\Phi}_{pe}$	0.025	1.25% liquidity premium, business loan
Production Sector			
Share of capital in production function	$\mu_c$	0.35	Standard
Share of pnt hhs' contribution in labor	$\alpha$	0.793	Iacoviello and Neri (2010)
Elast. of sub. in cons. good aggregator	$\epsilon$	10	Standard (Smets and Wouters, 2007)
Others			
Default cost fraction on output	$\zeta$	1	Standard
Income tax rate	$\tau_y$	0.204	Elenev (2017)

default  $\mu_m$  calibrated to 0.238. These parameter values can give a mortgage loan default premium around 0.5% (measured by the average spread between conventional mortgage rate and the MBS rate), as well as helping to match the share of mortgage credit in the financial intermediary's asset.<sup>50</sup>

<sup>50</sup> As seen in the model part, the financial intermediary holds two kinds of assets in the model economy, namely the mortgage loan and the entrepreneur or business loan. The data counterpart indicate about 3/4 for the weight of mortgage loan in this asset pool.

The mortgage contract and prepayment parameters are largely set according to work by [Greenwald \(2018\)](#). The mortgage amortization rate  $\varphi$  is set to 0.435% matching the average share of principal paid off on existing debt. The location and scale parameters  $\mu_\kappa$  and  $s_\kappa$  of the prepayment cost distribution are set to be 0.348 and 0.152, corresponding to an annualized prepayment rate of 14.98% in the steady state.

For the business sector, the entrepreneur population rate  $\gamma_e$  is set to 0.05.<sup>51</sup> The depreciation rate of capital is set to 0.025, as standard in the literature, implying a 10% depreciation rate annually. I set the survival rate of the entrepreneur  $\gamma = 0.94$  indicating an expected lifespan of business around 16.7 quarters, which is in a valid range of past studies, including BGG, [Christiano et al. \(2014\)](#), and [Ferrante \(2018\)](#).

The entrepreneur default rate is set to 0.875%, indicating an annualized 3.5% failure rate in the business sector. This is a little higher than the literature, as 2% in [Christiano et al. \(2014\)](#), and 1.5% in [Ferrante \(2018\)](#), in order to obtain the default premium of business loans in my model to be 0.8% annually, which is in line with the average spread between BAA and AAA corporate bond rates. Also, the steady state entrepreneur default risk, the standard deviation of the idiosyncratic shock distribution,  $\bar{\sigma}_e$ , and monitoring cost  $\mu$  are set to 1.25 and 0.7 respectively, to achieve the default premium mentioned above, as well as the share of business loans in the financial intermediary's asset holding.

For the financial intermediation sector, I calibrate the capital requirement ratio  $\nu_b = 0.09$  following [Gerali et al. \(2010\)](#). The steady state value of the intermediation cost variables,  $\bar{\Phi}_{pm}$  and  $\bar{\Phi}_{pe}$ , are set to 0.007 and 0.025 separately to match the liquidity premia of mortgage and business loans. They are 1% and 1.45%, based on the MBS and AAA spreads to the risk-free rate.

The share of capital in the intermediate goods producer's production function is  $\mu_c = 0.35$ , a standard value in the literature. The patient household's share of labor in the intermediate goods production is  $\alpha = 0.793$ , as the estimated value in [Iacoviello and Neri \(2010\)](#). The elasticity of substitution between retailers' goods in the final consumption good aggregator is calibrated to 10, following [Smets and Wouters \(2007\)](#). The income tax rate  $\tau_y$  is set to 0.204 following [Elenev \(2017\)](#). The dead-weight loss impact parameter  $\zeta$  is set to 1 in the benchmark case, as mentioned earlier.

### 3.3 Prior Distributions

The estimated parameters are listed in the table [2a](#) and [2b](#). I select the prior mean of the Calvo stickiness parameter  $\theta$  to be 0.67, implying a three-quarter frequency re-optimizing retailing price. The indexation parameter  $\iota$  is chosen to have a prior mean of 0.5, which gives no favor to either complete or no indexation. Given they take values on the unit interval, both these two parameters follow beta distributions with a standard deviation of 0.2. The adjustment cost parameters of capital and the financial intermediary's net worth,  $\phi_k$  and  $\phi_b$ , are set to follow a gamma distribution with mean and standard deviation of

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<sup>51</sup>This parameter does not affect the model steady state. In general, other model parameters are calibrated so that the average percentage of non-residential investment in output (from 1997Q1) is around 12.7%.

Table 2a: Estimation Results (structural parameters)

Description	Parameter	Prior			Posterior			
		Dist.	Mean	Stdev.	Mean	Stdev.	5 percent	95 percent
Price rigidity	$\theta$	beta	0.667	0.2000	0.1001	0.0497	0.0452	0.1844
Price indexation	$\iota$	beta	0.500	0.2000	0.392	0.1958	0.0622	0.6802
Adj. cost capital	$\phi_k$	gamm	10.000	5.0000	3.060	0.8873	1.7559	4.4670
Adj. cost FI net worth	$\phi_b$	gamm	10.000	5.0000	22.718	7.4924	11.2743	35.1334
Adj. cost FI portfolio	$\phi_s$	gamm	10.000	5.0000	6.154	0.7325	4.9719	7.3743
Credit supply cost elast., hhs. default	$\theta_d$	gamm	1.500	0.5000	1.318	0.0698	1.2075	1.4300
Credit supply cost elast., hhs. prepay	$\theta_p$	gamm	1.500	0.5000	1.537	0.5175	0.7049	2.3243
Credit supply cost elast., entrep. default	$\xi_d$	gamm	1.500	0.5000	2.489	0.2757	2.0380	2.9385
Persistence, losses on mortgage loan	$\rho_{lm}$	beta	0.300	0.1000	0.944	0.0092	0.9285	0.9586
Persistence, losses on business loan	$\rho_{le}$	beta	0.300	0.1000	0.869	0.0075	0.8567	0.8809
Capital utilization	$\psi_k$	beta	0.500	0.1500	0.889	0.0516	0.8141	0.9672
Consumption habit, imp hhs	$\eta$	beta	0.500	0.0750	0.647	0.0300	0.5991	0.6960
Consumption habit, pnt hhs	$\eta'$	beta	0.500	0.0750	0.263	0.0436	0.1920	0.3328
Interest rate response to inflation, MP	$\rho_\pi$	norm	1.500	0.1000	1.803	0.0772	1.6785	1.9314
Interest rate response to output, MP	$\rho_y$	norm	0.000	0.1000	-0.044	0.0538	-0.1232	0.0562
Lagged interest rate response, MP	$\rho_r$	beta	0.600	0.1000	0.230	0.0522	0.1453	0.3161

10 and 5, reflecting a less strong belief of the parameters' values. Habit parameters  $\eta$  and  $\eta'$  follow a beta prior with mean 0.5 and standard deviation 0.0075. For the conventional monetary policy parameters, I follow the literature and set the prior mean of the inflation reaction parameter to be 1.5. The prior means of output and interest lag responses are set to zero and 0.6 based on previous studies such as [Leeper et al. \(2017\)](#) among others.

The model estimates several novel parameters relating to the financial intermediation sector. To recap, the parameter  $\phi_s$  captures the adjustment cost for the FI changing its portfolio,  $\theta_d$ ,  $\theta_p$  and  $\xi_d$  depict relevant risk influencing the cost of the intermediation process, such as securitizing, monitoring and transacting these loans. I set all of their priors to be a gamma distribution with mean and standard deviation being 10 and 5, as a relatively less informative description. The persistence of the FI's unexpected losses follows a beta prior with mean and variance of 0.3 and 0.1, representing a weak belief of persistent losses.

The remaining parameters estimated describe the dynamics of exogenous shocks. I use an inverse gamma prior for the standard errors of the shocks, with a mean around 0.001 (0.1%) and a standard error of 0.01. For the shock persistence, I choose the beta prior with mean and variance of 0.8 and 0.1 respectively, following the work by [Iacoviello and Neri \(2010\)](#) among others.

### 3.4 Estimation Findings

The last four columns of Table 2a and 2b report the means, standard deviations, and 90% double-tail credible sets for the estimated parameters.

For the exogenous shock processes, the persistences are in general very high, with the lowest being 0.64 and highest 0.99. As to the standard deviations, the one with the housing demand shock is 11.6 percentage points. Given the model setup only features the

Table 2b: Estimation Results (the shock processes)

Description	Parameter	Prior			Posterior			
		Dist.	Mean	Stdev.	Mean	Stdev.	5 percent	95 percent
AR(1), productivity	$\rho_a$	beta	0.800	0.1000	0.837	0.0230	0.7997	0.8742
AR(1), housing demand	$\rho_j$	beta	0.800	0.0900	0.846	0.0309	0.7937	0.8950
AR(1), mortgage loan risk	$\rho_{\sigma_m}$	beta	0.800	0.1000	0.999	0.0008	0.9975	0.9997
AR(1), business loan risk	$\rho_\sigma$	beta	0.800	0.1000	0.993	0.0032	0.9879	0.9981
AR(1), investment	$\rho_i$	beta	0.800	0.1000	0.893	0.0270	0.8479	0.9352
AR(1), time preference	$\rho_z$	beta	0.800	0.1000	0.778	0.0412	0.7095	0.8436
AR(1), monetary policy	$\rho_e$	beta	0.500	0.1500	0.626	0.0447	0.5559	0.7002
St.dev., productivity	$\sigma_{ac}$	invg	0.001	0.0100	0.011	0.0009	0.0097	0.0127
St.dev., housing demand	$\sigma_j$	invg	0.001	0.0100	0.116	0.0089	0.1016	0.1301
St.dev., price markup	$\sigma_p$	invg	0.001	0.0100	0.304	0.1264	0.0745	0.4560
St.dev., mortgage loan risk	$\sigma_{\sigma_m}$	invg	0.001	0.0100	0.161	0.0176	0.1332	0.1897
St.dev., business loan risk	$\sigma_{\sigma_e}$	invg	0.001	0.0100	0.028	0.0020	0.0243	0.0309
St.dev., investment	$\sigma_i$	invg	0.001	0.0100	0.031	0.0048	0.0238	0.0387
St.dev., time preference	$\sigma_z$	invg	0.001	0.0100	0.020	0.0021	0.0167	0.0234
St.dev., monetary policy	$\sigma_e$	invg	0.007	0.0100	0.004	0.0005	0.0034	0.0050
St.dev. LTV imp hhs	$\sigma_\theta$	invg	0.001	0.0100	0.186	0.0143	0.1622	0.2086
St.dev., business loan premium	$\sigma_{rp}$	invg	0.001	0.0100	0.048	0.0039	0.0416	0.0540
St.dev., prepayment cost dispersion	$\sigma_\kappa$	invg	0.001	0.0100	0.506	0.0385	0.4419	0.5700

impatient households as marginal buyers of the house, this result shows the importance of the demand shock driving the business cycle fluctuations, echoing the results in the past studies; see, for instance, [Iacoviello and Neri \(2010\)](#), [Liu et al. \(2013\)](#), and [Iacoviello \(2015\)](#).

As to the loan contract related parameters, the loan loss persistences are estimated to be quite high (0.94 and 0.87 at the posterior mean), showing the loss impact the financial intermediary takes is rather significant. This finding could also indicate alternative sources of loan losses other than the forecast error loss this paper suggested. A more micro-founded mechanism may be needed to explain this persistence.

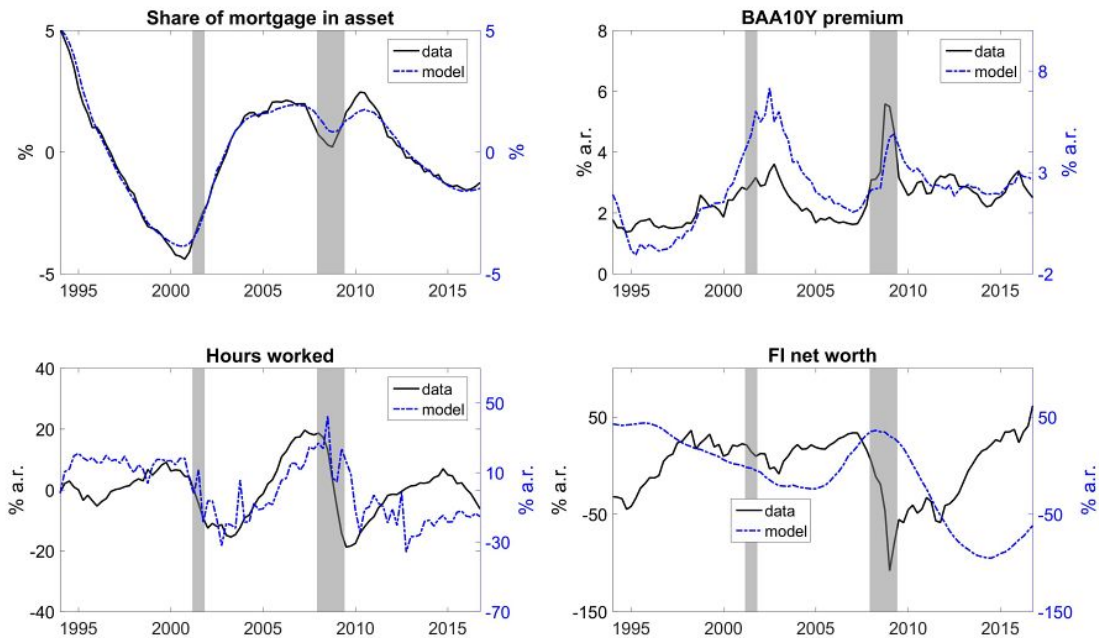
For the elasticities of FI's supply costs, those for the prepayment and default of households are estimated to be 1.31 and 1.53 respectively at the posterior mean, and the 90% credible set lower bounds are 1.21 and 0.70. The supply cost elasticity for the business loan is estimated to be 2.49 at posterior mean with the 90% credible interval between 2.04 and 2.94. These results indicate non-trivial credit supply cost variations influenced by borrowers' behaviors. Importantly, the portfolio adjustment cost parameter is estimated to be moderately positive with its posterior mean as 6.2. The result indicates that the financial intermediary is subject to certain barriers adjusting portfolio composition in reality, such as the loan decision-making time and effort taken in commercial banks. In the discussion below, I show the presence of this friction impedes the efficiency of the conventional monetary policy under a housing market downturn.

## 4 Model Mechanisms

### 4.1 Model Validation

To test the empirical fit of the model, I conduct an in-sample validation in order to test the empirical fit of the model. This exercise assesses the ability of the model to fit the data, especially for those time series not used as observables in the estimation.

Figure 4: In-sample Validation



*Notes:* The black solid lines represent actual data. Other than the BAA spread, all other series are deviation from their trends. The blue dashed lines represents the simulated data from the model.

Figure 4 shows the comparison results for four variables. The model time series are simulated based on the estimated shocks. As one chosen observable with observation errors, the mortgage share in total assets generated from the model tracks almost perfectly its data counterpart, showing the capability of model estimation fitting the actual data.

The other three variables in the figure are not used as observables. The second subplot shows the BAA spread (also shown in Figure 1). The model generally tracks the movement in the data and correctly captures the two peaks in recessions, although the peak in the early 2000s is larger than its data counterpart. This result indicates a reasonable impact of the default margin on the business loan spread implied by the model. The bottom left panel plots the hours worked series.<sup>52</sup> The model missed the dynamics of this variable at

<sup>52</sup>The data counterpart of hours worked in this subplot is constructed as hours from the nonfarm business sector (<https://fred.stlouisfed.org/series/HOANBS>) divided by the population series built in the appendix. The series is then quadratically detrended to retrieve the cyclical component.

the beginning and the end and performs better around the recession periods. However, in general, the simulated time series track the data counterpart reasonably well, given that the model has a relatively simplified block regarding the labor market.

The last panel shows the financial intermediary's net worth variable from the model, and its data analog represented by the banking sector index shown in Figure 1. The model has success tracking the magnitude and behavior of the fluctuation; however, the timing is off. As clearly seen, the net worth decline in the last recession is around four to five quarters late in the model compared to the data. The timing of the recovery in the financial sector is off by even more: the starting points of the two increases observed in the data (around 2003 and 2009) happen three and six years later in the model.<sup>53</sup>

There are several possible reasons for this observation. The losses to the financial intermediary are assumed to be persistent in the model, and the estimated parameters governing the persistence are close to unity. Therefore, the contemporaneous loss in the current period lasts for longer horizons. Another reason is that the losses in the financial sector used as observables lag the actual realizations of these losses due to the accounting practice in reality. The data counterpart used to represent the financial intermediary's net worth in the plot, on the other hand, is a banking sector index in the open market supposed to exhibit informational efficiency. Therefore, the banking sector index moves almost instantaneously given the fundamental changes in the economy, while the losses in the financial sector are observed in a later time. Also, this result may indicate that the model lacks an endogenized mechanism capturing bank run, which is a primary reason leading to the Great Recession.

## 4.2 Source of Fluctuation

For this subsection, I examine the source of the macroeconomic variables' fluctuations based on the proposed framework.

Figure 5 shows selected variables' historical decompositions. The mortgage risk shock drives mostly the change in the real house price and also the mortgage rate spread. The households' loan-to-value ratio disturbance, which is a popular candidate shock in the literature, also accounts for a significant portion of the house price and mortgage spread variation. The housing preference shock (not plotted) is not as important driving the fluctuations of these variables compared to previous studies, mainly due to the limited flow of housing stock in the current framework. These observations indicate the importance of the financial frictions in the household sector generating fluctuations in the key sectoral variables.<sup>54</sup>

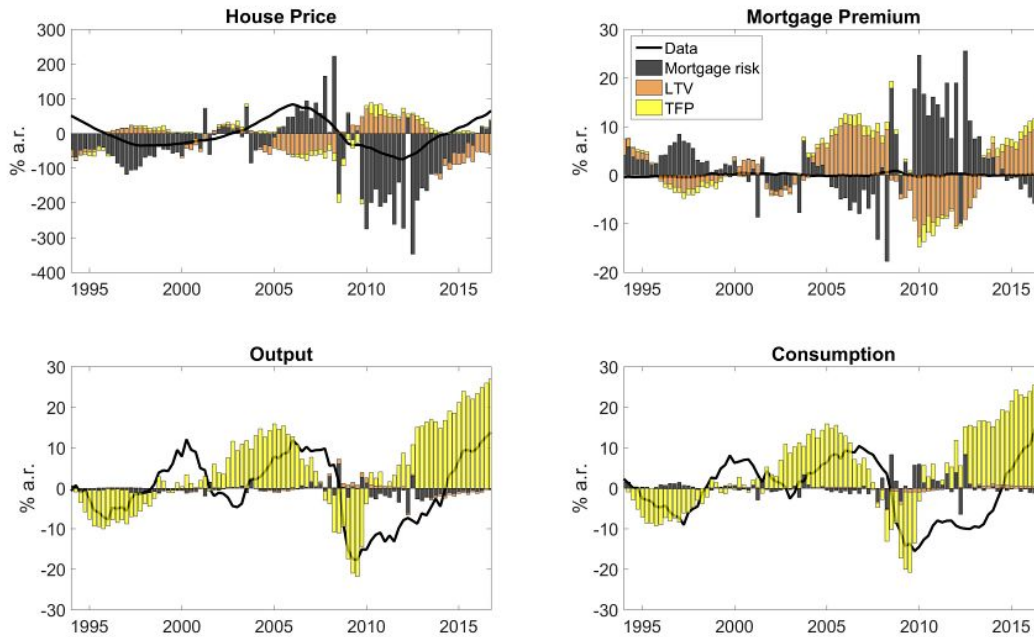
However, as can be seen in the third and fourth subplot, the aggregate output and consumption are only marginally influenced by the shocks relating to the financial frictions in the household sector. The primary driver of these two variables are still total factor

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<sup>53</sup>A very similar result also can be found in [Iacoviello \(2015\)](#), Fig.4.

<sup>54</sup>Notice that the loan-to-value ratio shock seems to drive the variables in the opposite direction as the mortgage risk shock. Later in the liquidity shock hypothesis section, I examine the property of this shock and show that it is subject to the critique by [Shi \(2015\)](#).

Figure 5: Historical Decomposition of the Estimated Model: Selected Variables and Shocks



productivities, given the current model allows the financial intermediary to share the aggregate risks with the borrowers and can suffer losses when the economy encounters a downturn. This result is opposite to the study by [Iacoviello \(2015\)](#) and favors the claim by [Suh and Walker \(2016\)](#) who argue that the financial frictions in the borrower sectors have limited power explaining the financial crisis in a linearized and estimated model through the financial intermediary's balance sheet.

### 4.3 The Prepayment and Default Incentives

In this subsection, I decompose and interpret the impatient household's optimality conditions for the prepayment and default thresholds. These decompositions facilitate understanding the forces driving the changes in these rates.

**The Prepayment Incentive.** The model gives a clear structural breakdown of refinancing incentives. To see this, I rearrange the first order condition for the prepayment threshold



into the following form

$$\kappa_t^* = \underbrace{(1 - \Omega_t^m - \Omega_t^x r_{m,t-1}) \left( 1 - \frac{(1 - \varphi) \pi_t^{-1} m_{t-1}}{m_t^*} \right)}_{\text{New debt incentive}} - \underbrace{\Omega_t^x (r_{m,t}^* - r_{m,t-1})}_{\text{Interest incentive}} - \underbrace{(q_t^h - \Omega_{t-1}^h)(h_t^* - \tilde{h}_t) / m_t^*}_{\text{Price incentive}} \quad (42)$$

where  $\tilde{h}_t = \frac{1 - G_{m,t}}{1 - \Delta_t} h_{t-1}$  and can be interpreted as the average housing stock left after foreclosure (prior to the refinancing);  $\Omega_t^m$  and  $\Omega_t^x$  represent the marginal costs of increasing the mortgage and promised payment respectively.<sup>55</sup>  $\Omega_t^h$  is the marginal value of increasing one unit of housing stock at  $t$  for the impatient household. Now it can be seen that the prepayment or refinancing rate  $\varrho_t = \Gamma_{\kappa^p}(\kappa_t)$  is determined by the following three ingredients.<sup>56</sup>

The first term denotes the benefit of taking on new debt, normalized by the scale of the new debt, and rescaled by the net benefit excluding the cost of continuing debt and promised payment. The second term indicates the impatient household's comparison between the new mortgage loan coupon rate and the existing rate, scaled up by the cost of promising the payment. These two incentives are discussed in [Greenwald \(2018\)](#). The third term reflects the price incentive. The term  $\Omega_t^h$  can be interpreted as the implicit price of the housing stock for the impatient households.<sup>57</sup> Notably, the existence of default in the model makes the steady state new housing size less than the current stock, meaning  $h^* < \tilde{h}$ , and this can hold in the neighborhood when considering model dynamics. So when the gap between the market price for housing ( $q_t^h$ ) and the subjective evaluation of households ( $\Omega_t^h$ ) widens, the incentive to refinance for a new place to live increases. On the other hand, possibly, if the newly financed housing stock to live is more than the current one, i.e.,  $h_t^* > \tilde{h}_t$ , a higher house price than subjective evaluation will lead to a decrease in prepayment rate. This effect is scaled by the new debt level  $m_t^*$ .

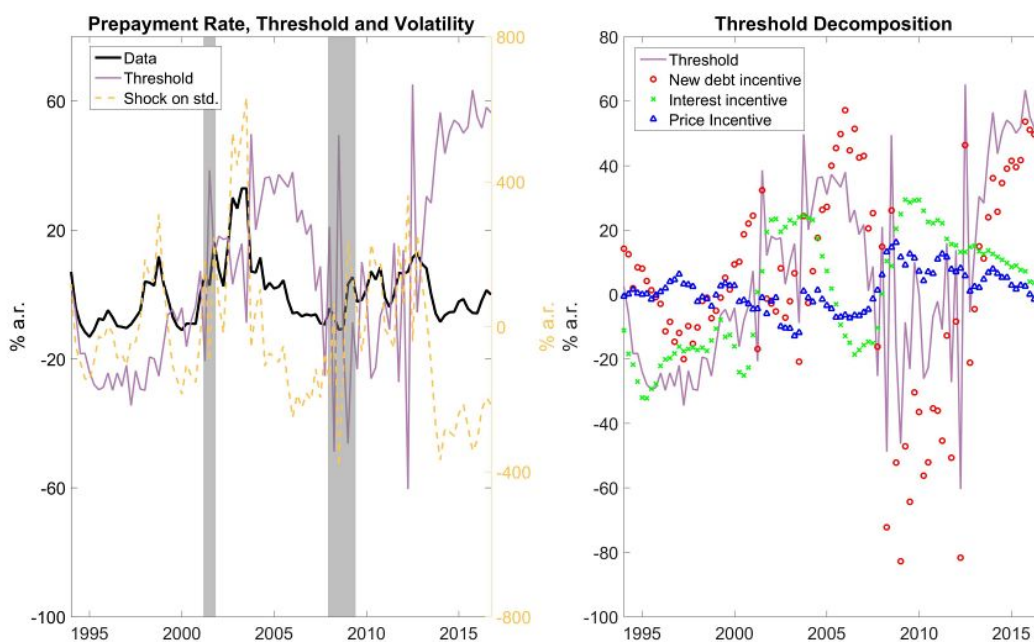
As an application of the theoretical framework, I look into the prepayment rate according to the derivation above to see how these incentives drive the actual data. [Figure 6](#) shows the decomposition the prepayment rate observed in the data. The left panel plots the actual data in the solid black line, which is driven by two variables in the model, namely the prepayment threshold (represented by the light color solid line) and the exogenous shock on the standard deviation of the prepayment cost distribution (represented by the light color dotted line). The shock to volatility overall tracks the movement of the actual prepayment rate data, albeit with its change prominently larger in magnitude, re-

<sup>55</sup> $\Omega_t^m$ ,  $\Omega_t^x$ , and  $\Omega_t^h$  are essentially Lagrangian multipliers from the impatient household's optimization problem. The details can be found in the appendix.

<sup>56</sup>Here I use the fact that the refinancing rate is a monotonic function of the threshold policy variable  $\kappa_t^p$ , which is from the property of a standard c.d.f. Therefore, a higher prepayment threshold value corresponds to a higher rate of prepayment. The same situation applies for the default rate as can be seen below.

<sup>57</sup>From FOCs later, one can see that  $\Omega_t^h < q_t^h$ , because of the heterogeneous preferences and the collateral constraint.

Figure 6: Decomposition of Prepayment Rate



flecting the model capturing the volatility of the prepayment rate variation in the corresponding structural shock term.

The movement pattern of the decision threshold, on the other hand, can deviate from the observed prepayment rate, especially when the dispersion of the prepayment cost distribution is small. This increase in the decision threshold can be observed between 2004 to 2008 before the recession, and after 2013 when the housing market starts to recover. To see further the forces driving the decision threshold, the right panel of Figure 6 shows the series of three prepayment incentives generated from the estimated model.

The new debt incentive is the most volatile series of the three, which also most closely tracks the changes in the prepayment threshold. This incentive is primarily responsible for the surges in the prepayment decision threshold mentioned above. This observation indicates that during these two periods, households have higher incentives than other times to take on more debt and refinance their mortgage. The incentive plummets and fluctuates significantly during and after the recession for around four years, corresponding to the undesirable fundamentals during this period.

The other two incentive series are not as volatile overall. The interest incentive (marked by cross in the plot) generally moves opposite to the mortgage spread and the policy rate: it can be clearly seen that this incentive is low prior to the latest recession as the benchmark interest rate was high by then and it is not worthwhile to refinance one's mortgage under this high rate. The incentive is high after the crisis corresponding to the close-to-zero policy interest rate in that period, as well as before 2005 in which year the interest rate starts rising. Lastly, the price incentive moves relatively less than the other two, possibly

because of its relation to the change in the households' housing stock held before and after refinancing, which is quite small. However, it is obvious that this incentive is low during the housing price boom before the latest recession and prominently increases during and after the crisis. This observation shows that the current market price of the house plays a role determining the households' prepayment decisions.

**The Default Incentive.** Similar to the prepayment case, the first order condition for the default threshold  $\omega_{m,t}^*$  can also be grouped into several terms describing the agent's incentive to default as

$$\begin{aligned}
\omega_{m,t}^* = & \underbrace{\left( \left( \varrho_t + (1 - \varrho_t) \frac{\Omega_t^h}{q_t^h} \right) G_{q^h,t} \right)^{-1}}_{\text{price effect of asset holding}} \theta_{t-1}^{LTV} \cdot \left\{ \underbrace{\frac{1 + (1 - \tau_y)r_{t-1}}{\pi_t}}_{\text{real interest payment net of tax deduction}} \right. \\
& - \underbrace{(1 - \Omega_{m,t} - \Omega_t^x r_{m,t-1}) \left( \varrho_t \frac{m_t^*}{m_{t-1}} + (1 - \varrho_t)(1 - \varphi)\pi_t^{-1} \right)}_{\text{debt holding effect}} \\
& \left. + \underbrace{\varrho_t \Omega_t^x \left( r_{m,t}^* \frac{m_t^*}{m_{t-1}} - r_{m,t-1} \right)}_{\text{interest effect under refinancing}} + \underbrace{\varrho_t \left( 1 - \frac{\Omega_t^h}{q_t^h} \right) \frac{m_t^*}{m_{t-1}} \frac{1}{\theta_t^{LTV*}}}_{\text{price and leverage effect under refinancing}} \right\} \quad (43)
\end{aligned}$$

where  $G_{q^h,t} = q_t^h / q_{t-1}^h$  is the real house price growth rate. In the model, the impatient households make the default decision prior to prepayment, thus they take into account the refinancing possibility when making a decision to default. The contemporaneous default rate  $\Delta_t = F_t^m(\bar{\omega}_{m,t})$  is thus determined by the following factors.

The scaling term outside the curly bracket of equation (43) is a common factor that signals the price effect of the housing asset on mortgage default. The first part in the first term is essentially a house price growth measure weighted by the prepayment expectation of the impatient households. The higher this asset price increase is compared to the previous period, the less likely the borrower will default. This price effect is scaled by the leverage of the household, as a lower leverage level (a higher net worth of the housing stock) will scale up this price effect, and vice versa.

In the curly bracket, the first term is the real interest rate payment net of the tax deduction incentive, and the higher this promised interest payment is, the more likely the mortgage borrower defaults. The second term shows the debt holding effect: the first part in the parenthesis depicts the net benefit of holding outstanding debt (as discussed in the prepayment incentive part), and the latter part is an expected stock of debt with respect to refinancing possibilities. Intuitively, the higher outstanding debt the borrower can hold, the better the situation they are in, since they do not need to pay it back immediately.

The third and last terms in the curly brackets are both conditional on the situation of prepayment. The former increases the likelihood of default if the newly financed mortgage has a higher coupon payment burden than the old contract. The latter indicates that

the more new housing costs (comparing to the borrower's subjective evaluation), the more debt the borrower accrues, while the less the household can contemporaneously borrow against its new housing stock (lower  $\theta_t^{LTV*}$ ), the more likely the borrower will default.<sup>58</sup>

#### 4.4 The Portfolio Choice Mechanism

As a novel feature presented by this paper, I here show the mechanism that determines the loan portfolio share of the financial intermediary. From its related first order conditions, I can get the expression for the share of mortgages in the current period as

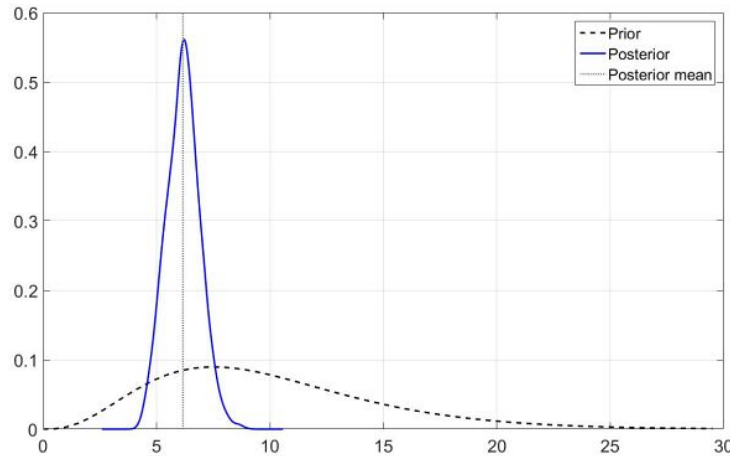
$$s_t = \frac{\tilde{r}_{m,t}^{I,a} - \tilde{r}_{e,t}^{I,a} + \Phi_{pe,t} + \phi_s s_{t-1}}{\Phi_{pm,t} + \Phi_{pe,t} + \phi_s}. \quad (44)$$

Log-linearization of this equation yields

$$\hat{s}_t = \frac{\phi_s \bar{s}}{\mathcal{A}} \cdot \hat{s}_{t-1} + \frac{\bar{r}_m^{I,a}}{\mathcal{A}} \cdot \hat{r}_{m,t}^{I,a} - \frac{\bar{r}_e^{I,a}}{\mathcal{A}} \cdot \hat{r}_{e,t}^{I,a} + \left(1 - \frac{1}{\mathcal{B}}\right) \cdot \hat{\Phi}_{pe,t} - \frac{1}{\mathcal{B}} \cdot \hat{\Phi}_{pm,t} \quad (45)$$

where  $\mathcal{A} = \bar{\Phi}_{pe} + \phi_s \bar{s} + \bar{r}_m^{I,a} - \bar{r}_e^{I,a} > 0$ , and  $\mathcal{B} = \bar{\Phi}_{pm} + \bar{\Phi}_{pe} + \phi_s > 0$ . Variables with bars stand for its deterministic steady-state value.

Figure 7: Prior and Posterior distributions of the portfolio adjustment cost parameter  $\phi_s$



As clearly seen in equation (45), the share of mortgage loans positively relates to its net real return  $\tilde{r}_{m,t}^{I,a}$  and negatively relates to its intermediation cost  $\Phi_{pm,t}$  and the alternative asset's return rate  $\tilde{r}_{e,t}^{I,a}$ . The  $\Phi_{pe,t}$  seems to have ambiguous influence of the mortgage share in asset as the sign of  $\left(1 - \frac{1}{\mathcal{B}}\right)$  is indeterminate, but given the posterior estimate of

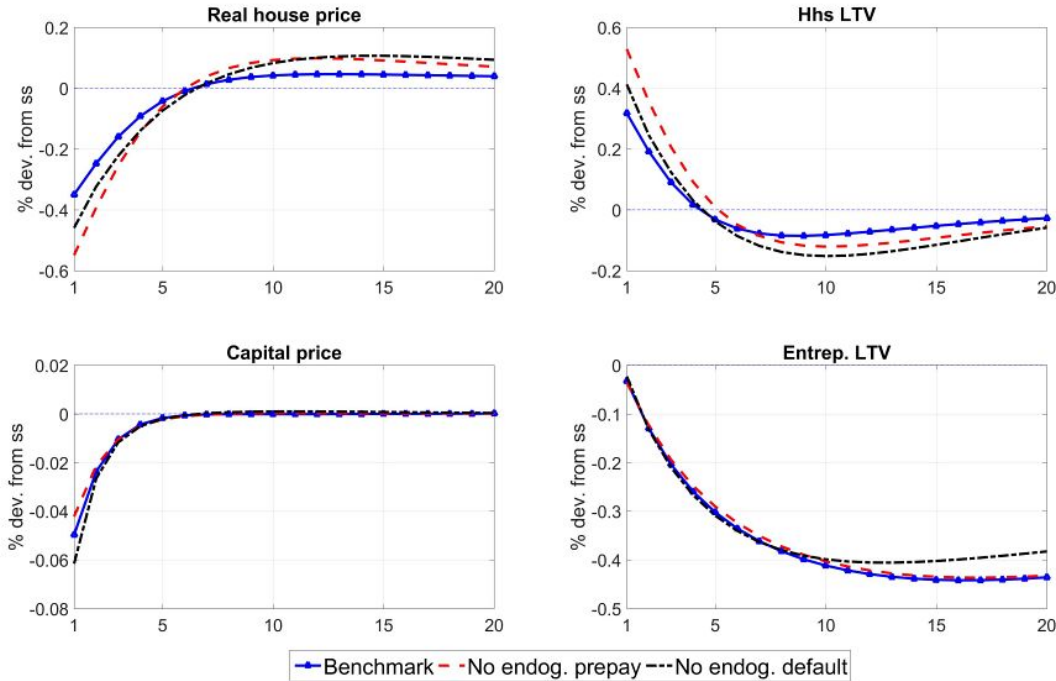
<sup>58</sup>All the terms in the curly bracket are normalized by the stock of debt in the previous period, i.e.,  $m_{t-1}$ .

$\phi_b$  shown below,  $\mathcal{B} > 1$ , and the intermediation cost of business loans moves to the same direction with the share of mortgage  $s_t$ , which is rather intuitive.<sup>59</sup>

## 4.5 The Liquidity Shock Hypothesis

This subsection briefly examines the relationship between the asset prices and related agents' leverages subject to various exogenous shocks. As stated in Shi (2015), the KM type of model can create negative comovements between asset prices and loan-to-value ratios (LTV) given liquidity and other types of shocks, which is counterfactual and violates the so-called liquidity shock hypothesis (LSH hereafter). The LSH describes the empirical fact that asset prices move in the same direction with credit availability. I examine the model's ability to account for this issue.

Figure 8: Liquidity Shock Hypothesis Test: Responses to 1% Business Loan Risk Increase



Notes: Other than parameters controlling the existence of endogenous default and prepayment, the impulse response functions are evaluated at the posterior means of the estimated parameters.

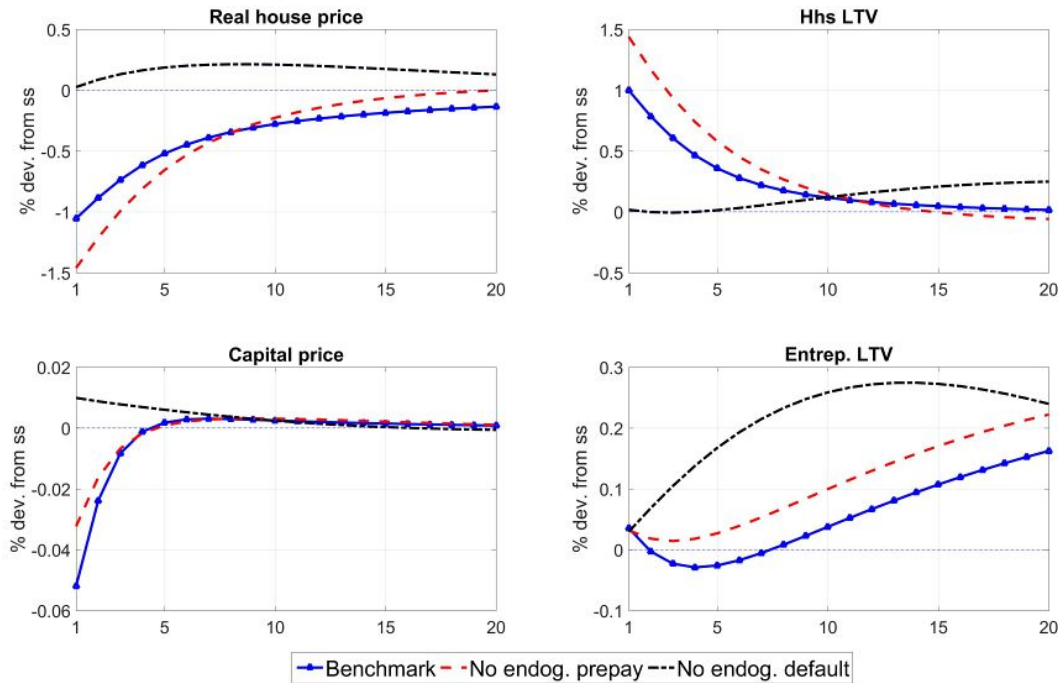
Figure 8 plots the responses of asset prices and LTVs to a positive business loan risk shock. Three cases are plotted: the benchmark economy where the model parameters are from the posterior mean estimates, and two other cases with endogenous prepayment and default of mortgage loans shut down respectively. I find that the capital price and

<sup>59</sup>Given the empirical result, this adjustment cost is reduced-form modeling of portfolio adjustment frictions in reality, which needs further study to be better understood. In the appendix, I also show analytically the influence of this portfolio share to the interest rate spreads.

the loan-to-value ratio can move together in all cases, suggesting that the business loan contract of the BGG form can generate positive comovement between the asset price and loan-to-value ratio under a loan default risk shock proposed by [Christiano et al. \(2014\)](#). The reason is that the risk shock weakens the entrepreneur's balance sheet strength by decreasing its net worth; meanwhile, the investment project also is less attractive; thus, the price of capital drops while the demand for investment decreases. This mechanism follows the general logic to resolve the LSH issue proposed by [Shi \(2015\)](#).

On the contrary, the households' average loan-to-value ratio and real house price have negative comovement. This fact is, in general, true for all shocks considered in the model.<sup>60</sup> This result verifies the conclusion drawn by [Shi \(2015\)](#) since the households' collateral requirement for new debt is set to be fixed as in KM and shocks leading to a shortage in liquidity can make the asset more valuable given other conditions unchanged, and vice versa. This observation shows that the determination of the loan-to-value ratio in the economy needs to be further understood, and an endogenized contract might be needed to resolve the puzzle.

Figure 9: Liquidity Shock Hypothesis Test: Responses to 1% Households LTV ratio Increase



Notes: Other than parameters controlling the existence of endogenous default and prepayment, the impulse response functions are evaluated at the posterior means of the estimated parameters.

As another confirmation of the result, Figure 9 shows an exogenous loan-to-value ratio increase of the impatient households. As the collateral becomes less scarce, the asset price

<sup>60</sup>The plots for other shocks are in the appendix. One can observe that the asset price and the loan-to-value ratio always have symmetric, opposite movements.

drops. The co-movement of capital price and entrepreneur loan-to-value ratio is more ambiguous, however in general they still move in the opposite directions. The shock is a standard liquidity related shock, and these observations above are again subject to the critique by [Shi \(2015\)](#). Notice that the case without endogenous mortgage default can somewhat create positive co-movement between asset prices and corresponding borrower's loan-to-value ratio. However as I will show in the monetary policy section, it is in general counterfactual to have default mechanisms in the model with agents not responding to incentives endogenously.

## 5 Monetary Policy Implication

This section conducts monetary policy analysis, including conventional and unconventional ones, by employing the framework above.

### 5.1 Conventional Monetary Policy

For the effectiveness of conventional monetary policy, I consolidate the conclusion drawn by [Greenwald \(2018\)](#) that, under a deflationary TFP shock, the refinancing facilitates stabilizing inflation, while generating more volatility in household debt accumulation. The mechanism, named the 'mortgage credit channel',<sup>61</sup> can impose a trade-off for the monetary authority if the credit boom is a concern. I further find that the endogenous mortgage loan default activities can dampen this trade-off on the household sector, as default activities restrict the refinancing quantity and thus the new debt issued. At the same time, the business sector credit level reacts more. Moreover, I find that the monetary policy effectiveness depends on the source of the fundamental shock. The same channel above can reduce the effectiveness of the conventional monetary policy and may need stronger stimulus under a mortgage loan risk shock. Credit supply mechanisms proposed in this paper can further exacerbate this problem.

#### 5.1.1 TFP shock

To demonstrate the points above, I perform the experiment proposed by [Greenwald \(2018\)](#), where the central bank uses the interest rate rule to completely stabilize inflation (implying an infinite elasticity of interest rate to inflation change). Given the extremeness of the policy, this experiment tests the effectiveness of the interest rate policy straightforwardly: the less change is needed to stabilize inflation, the more effective the monetary instrument is.

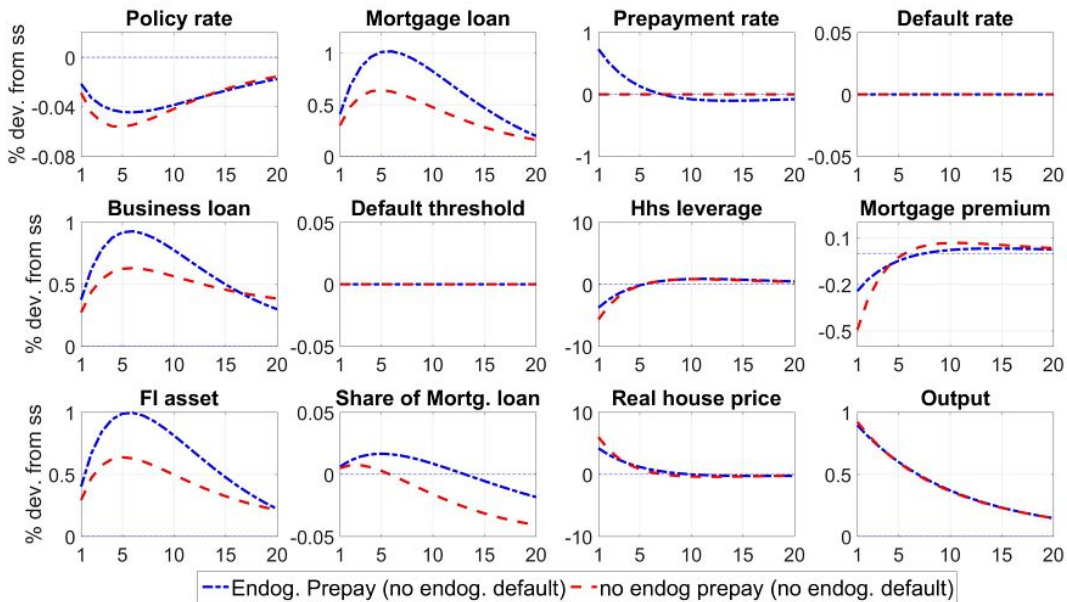
Figure 10a shows the impulse responses of two different scenarios given a 1% productivity increase. The dashed-dotted blue line and the dashed red line are for the cases with

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<sup>61</sup>The channel is defined in [Greenwald \(2018\)](#) as the propagation mechanism of fundamental shocks through mortgage credit issuance to the rest of the economy. In my study, I keep the prepayment feature of his model, and add an endogenous default mechanism as another attribute of this channel.

and without endogenous prepayment respectively, with the endogenous default choice shut down. Under this productivity increase, which is deflationary, the monetary authority has to decrease the interest rate to bring the inflation rate up to its target, and the interest rate cut lasts for an extended period given the persistence of the shock. Compared to the endogenous prepayment case, the economy with no internal refinancing mechanism needs a lower interest rate (at least on impact) to stimulate aggregate demand and maintain the inflation target. A smaller interest rate cut is needed when endogenous prepayment exists, since the households take advantage of the low interest rate and increase their mortgage debt by refinancing on impact. However, this credit boom generated by more refinancing activities also could be considered a side effect of the policy, imposing a potential trade-off to the monetary authority.

Figure 10a: Response to 1% Productivity Increase, Full Inflation Stabilization



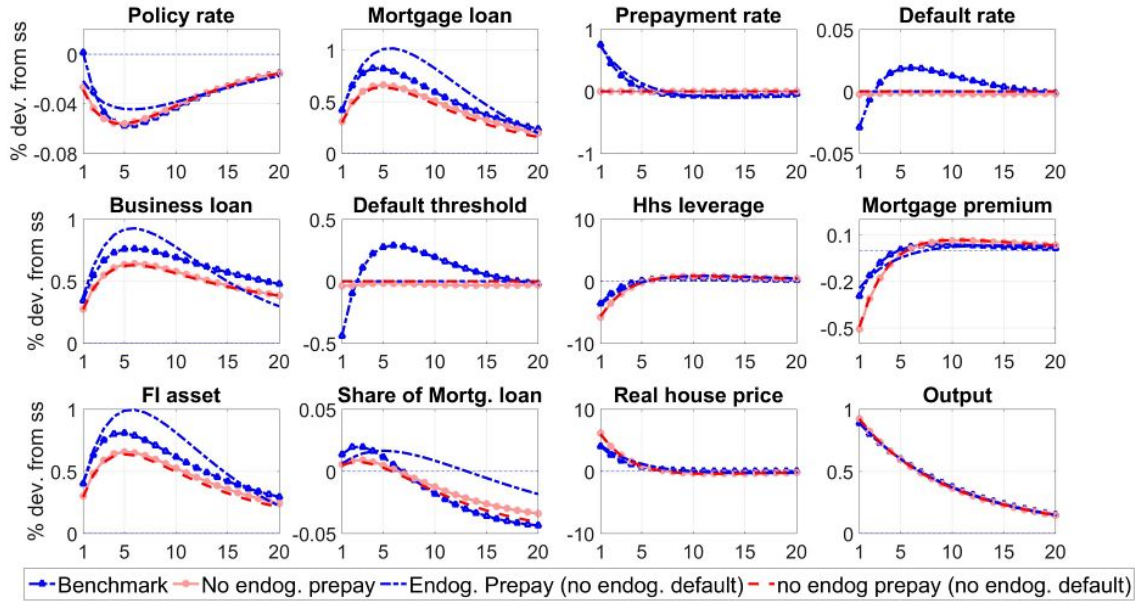
I find the trade-off between inflation stabilization and credit boom limitation dampens in the presence of endogenous default. Figure 10b plots two additional cases to figure 10a (the previous figure), including the benchmark economy (with all parameters fixed at the posterior mean estimates) and an alternative economy with exogenous prepayment rate (fixed), represented by the solid blue and red lines with triangles and circles respectively. Given the interest rate incentive, households take on new debt, which increases the price of the mortgage. The mortgage premium lowers on impact, but it increases with the default rate after the second quarter. As seen in the figure, the default rate (as well as the threshold) displays a hump-shaped increase similar to the mortgage premium change. As a consequence, fewer households prepay mortgage debt, and the credit demand is limited, leading to a smaller expansion in mortgage debt.

However, it is worth noticing that the business sector credit increases more than the two cases without endogenous prepayment, and the increase becomes the highest in all



cases after thirteen quarters. The rationale is that given the monetary authority has to cut the interest rate (to maintain the policy target) and the credit demand is limited on the household side (by a higher default rate), the business sector expands its credit more easily. On the supply side, the financial intermediary also substitutes loanable funds away from the household sector, since business loans become more profitable than mortgage loans as the productivity increases.

Figure 10b: Response to 1% Productivity Increase, Full Inflation Stabilization (cont.)



In general, the qualitative results still hold after introducing endogenous default, but quantitatively the effects are lessened significantly. The mortgage credit boom at its peak is about 20% less (82bps and 102bps) in the benchmark case compared to the situation with an exogenous default rate. The policy rate needs to be cut to a lower level in the benchmark economy, close to the cases with no endogenous prepayment after four quarters, albeit it is relatively high on impact and in the short run (within four quarters) because the initial household prepayment rate increases. Good news for the policymaker might be that the concern of the households' leverage increasing too much could be alleviated. However, the authorities may need to be aware that cutting the interest rate could be less effective, and a credit boom in the business sector can take place.

### 5.1.2 Mortgage Loan Risk Shock

A more efficient monetary policy with endogenous refinancing is not always the case. I further find that the influence of the mortgage credit channel on the monetary policy effectiveness can depend on the source of the shock. Here I consider the mortgage loan risk shock as it is the most relevant one for the latest recession.

Figure 11: Response to 1% Mortgage Loan Risk Increase, Full Inflation Stabilization

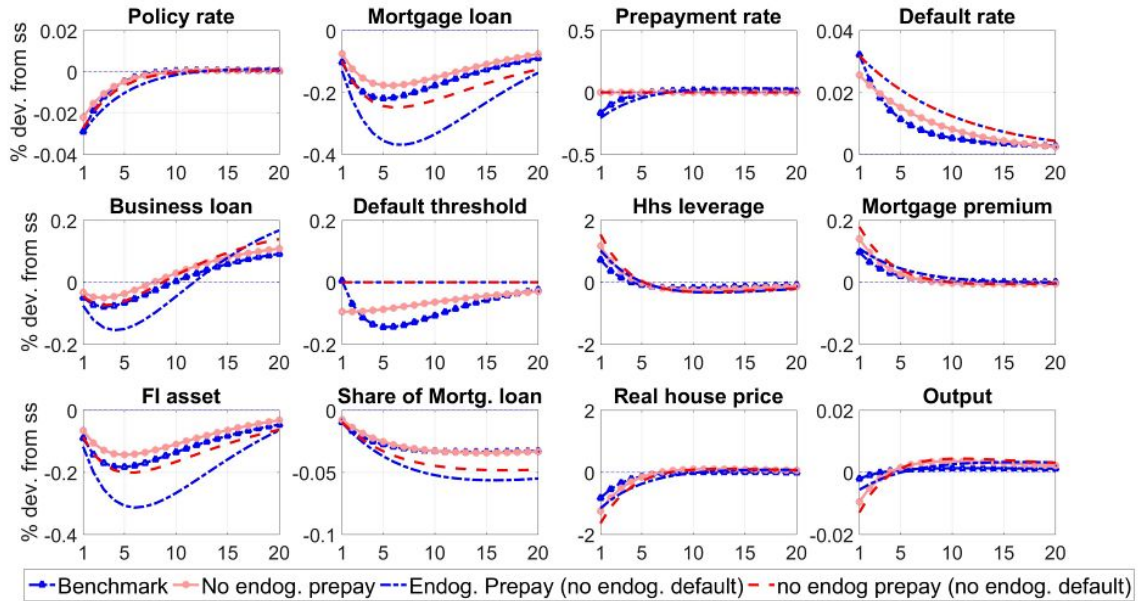


Figure 11 shows the responses with a 1% mortgage risk shock increase, repeating the four cases considered with a TFP shock above. With endogenous prepayment, the households reduce the new debt issued, and thus the stock of liability born, limiting aggregate demand. Also, being qualitatively similar, the quantitative impact is in general dampened by the endogenous default, as the agent chooses to lower the default threshold to prevent further damage to their net worth, and this leads to less variation in household liabilities. As a consequence, the interest rate needs to be more than 30% lower on impact to achieve the policy target in the benchmark case compared to the scenario where only the prepayment rate is exogenous (2.89 bps vs. 2.21 bps decrease). For the two cases without endogenous default, although the interest rate cuts are similar on impact, the economy with endogenous prepayment has a persistently lower interest rate for at least ten quarters compared to its counterpart with exogenous refinancing, due to the deep contraction in mortgage loan quantity.

Overall, the analysis indicates that under a mortgage loan risk shock, a deeper interest rate cut may be needed when the mortgage credit channel exists as the mortgage loan risk increases, albeit the endogenized default decisions could dampen this impact. In general, this result holds for any shock that deteriorates the households' financing conditions, such as a shock decreasing the financial intermediary's net worth, while for shocks which make refinancing more desirable and increase the debt stock, conventional monetary policy becomes more efficient, such as a business loan risk shock.<sup>62</sup> The mechanism can serve as a possible explanation of the Federal Reserve sharply cutting the interest rate during the last recession. Meanwhile, it raises the likelihood of needing alternative policies as substitutes, and which is also what happened after the short-term interest rate reached the zero

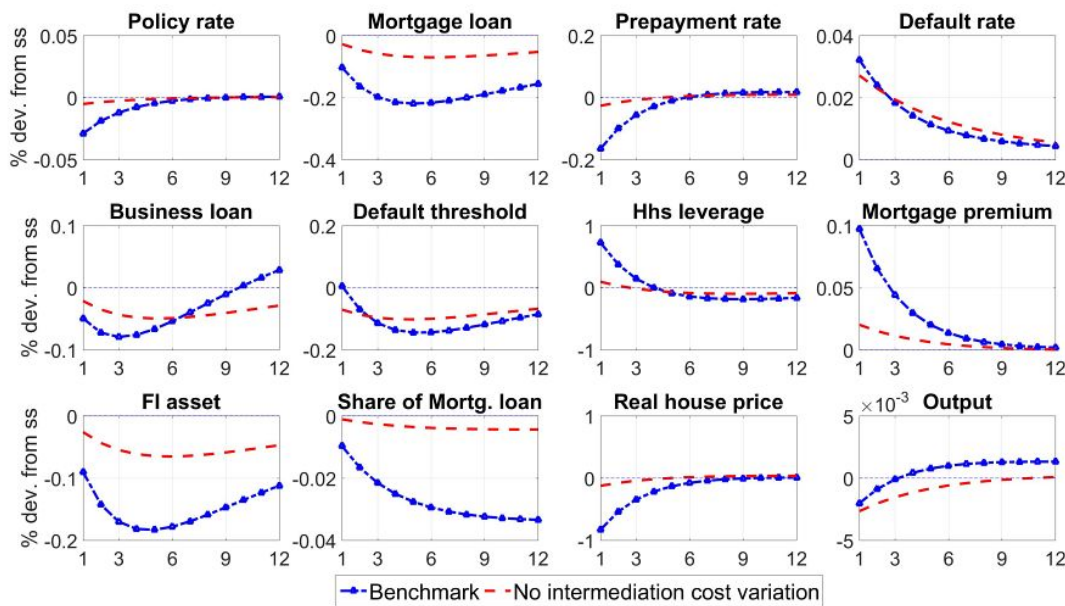
<sup>62</sup>I present the impulse response plots for these cases in the appendix.

lower bound.

**Role of Credit Supply Channels.** In this section, I examine if the channels on the credit supply side proposed in this paper play a role in affecting the monetary policy outcome. I conduct the comparisons under the mortgage loan risk shock for major interest, as it represents one of the major issues encountered in the last recession.

Firstly, I examine the role of intermediation cost variation with real sector activities. Figure 12a shows the impulses responses to a 1% mortgage loan risk shock under the benchmark economy and the economy with no intermediation cost variation.

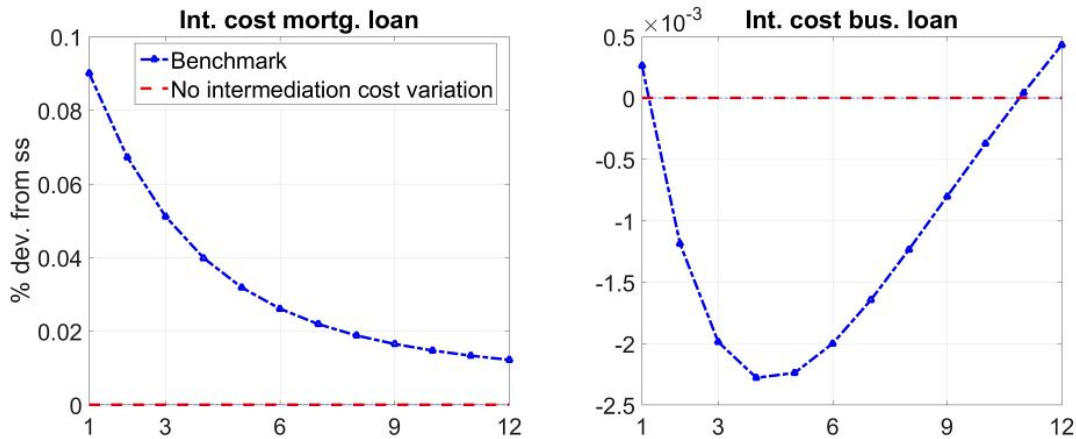
Figure 12a: Effect of Intermediation Cost Variation



The responses, in general, are more significant quantitatively in the benchmark model than its counterpart with invariant intermediation costs. The rise in the cost to intermediating mortgage loans drives up the corresponding loan premium and lowers the loan quantity supplied to the households. This change spills over to the business sector, as the intermediation cost for the business loan also increases after ten quarters. Given the presence of the portfolio adjustment mechanism, the financial intermediary adjusts the asset composition from mortgage loans to business loans, leading to an increase in the latter after the initial decline, however overall it decreases its total asset holding. As a result, the interest rate cut in the benchmark case is close to six times as much as the scenario without the intermediation cost variation (2.89 bps vs. 0.51 bps.)

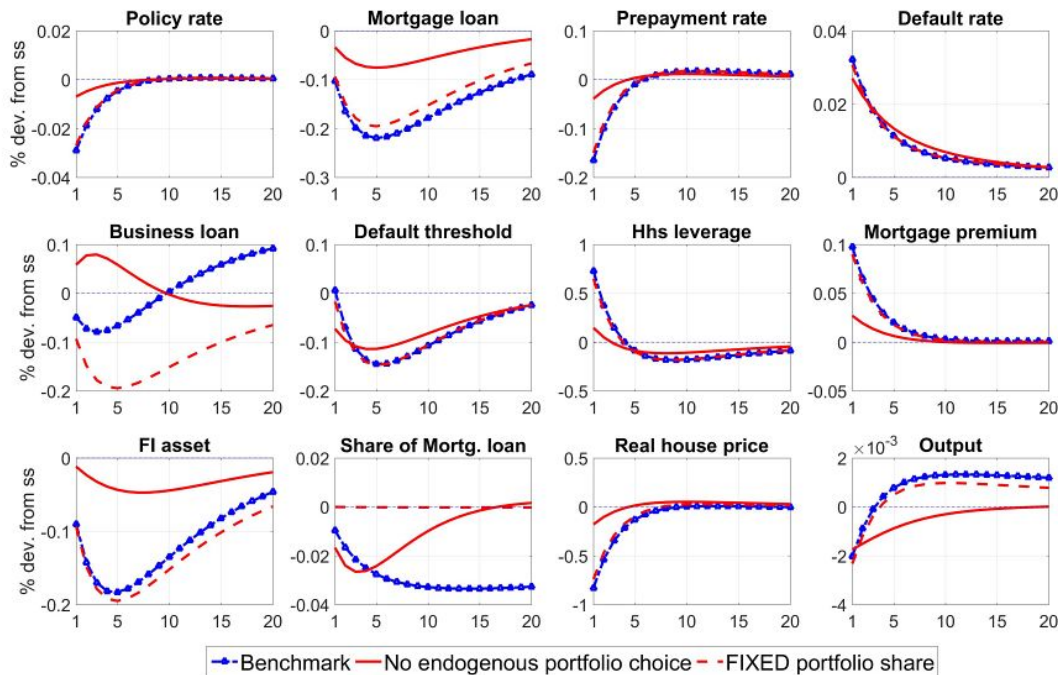
The other important mechanism incorporated in the paper is the endogenized portfolio choice of the financial intermediary. Figure 13 compares three cases: the benchmark case, the economy with no portfolio choice effect, and economy with fixed shares of assets held by the financial intermediary. When the endogenous choice between different assets is absent, the supply schedules of the financial intermediary are horizontal lines (perfectly

Figure 12b: Effect of Intermediation Cost Variation (cont.)



elastic), and only the demand side drives the equilibrium quantities of credit. The intermediation cost variation discussed above can shift the supply curves vertically; however, the ‘passive’ supply schedule make the flow of loanable funds easy between assets, and can wash away this cost variation substantially. As a result, the impulse responses, in this case, are very similar to the economy with fixed intermediation costs (as shown in Figure 12a), and the interest rate does not need to respond as much.

Figure 13: Effect of Financial Intermediary Portfolio Choice



The last scenario in figure 13 represents an extreme case where the cost of adjusting portfolio composition for the financial intermediation is infinite. Although it is pre-

sumably counterfactual, the demonstration gives a better understanding of the portfolio choice friction. This case in general generates responses close to the benchmark case with several exceptions. The business loans have a deep drop and the minimum is more than twice as much as the benchmark case (7.92 bps and 19.5 bps). The financial intermediary also decreases its asset holdings 6.6% more than the benchmark case at the minimum (18.31 bps and 19.52 bps). The intuition for the result is that, as the financial intermediary cannot substitute away from the asset suffering an intermediation cost hike, it has to decrease its total credit supply to lower the impact. The monetary policy still needs to react strongly in this scenario, but a little less than the benchmark case (the different is about 0.1 bpts).

## 5.2 Unconventional Monetary Policy

This subsection examines the effects of unconventional monetary policy, as such policies were widely adopted by central banks after the Great Recession. I show that the mortgage credit channel can enhance the effect of the policy considered in this experiment. Meanwhile, credit supply cost decreases are crucial for reinforcing the policy change, without which the policy outcome can be overturned. Portfolio adjustments of the financial intermediary help reallocate resources to the household sector, and marginally benefit the policy outcome.

Here, I model the unconventional monetary policy as an exogenous change in the mortgage loan rate, similar to [Pietrunti and Signoretti \(2017\)](#). The variation in this long-term rate corresponds to price changes of the underlying asset. For instance, as the market price of the mortgage loan contract increases, the associated interest rate drops. This mechanism corresponds to what has happened in the mortgage loan market during and after the last recession: as the mortgage market collapsed, the Federal Reserve conducted large-scale purchases of agency mortgage-backed securities (MBS hereafter) starting in late 2008.<sup>63</sup> This activity drove up the price of the mortgage loan and related assets by increasing the demand for MBS and lowered the corresponding yields or interest rates. Also for the experiments, I keep the short-term policy rate fixed to its steady-state level. This assumption corresponds to the situation that the monetary authority commits to a certain interest rate level, or has limited ability to change the short-term policy rate. The setup also can isolate the effect of the unconventional policy, making the observation and comparison clearer.<sup>64</sup>

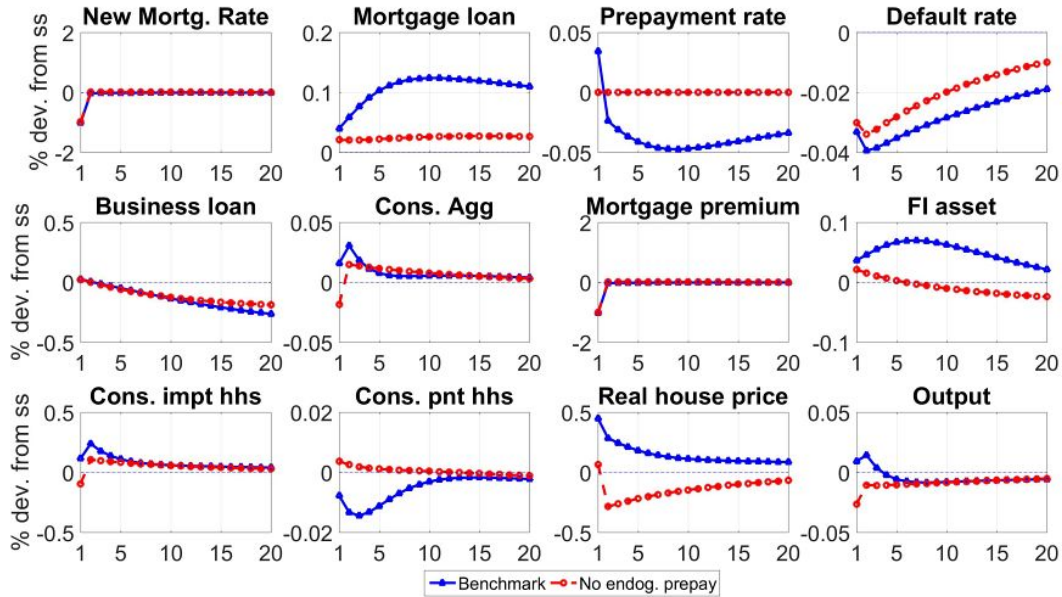
Figure 14 demonstrates the responses to a 1% decrease in the mortgage loan rate lasting for one quarter, with the blue line with triangles representing the benchmark case and the red dashed line with circles as the economy without the endogenous prepayment mechanism. In both cases, the default rates of mortgage loans decrease as the lowered interest rate alleviates household pressure to honor their debt. This effect is long-lasting due to the long-term property of the mortgage loan contract.

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<sup>63</sup>See [Ferrante \(2018\)](#) and [Elenev \(2017\)](#) for detailed discussions of the background.

<sup>64</sup>To fix the short-term rate, I set the persistence of the lagged interest rate in the Taylor rule close to one, following [Pietrunti and Signoretti \(2017\)](#).

Figure 14: Response to 1% Mortgage Loan Rate Decrease



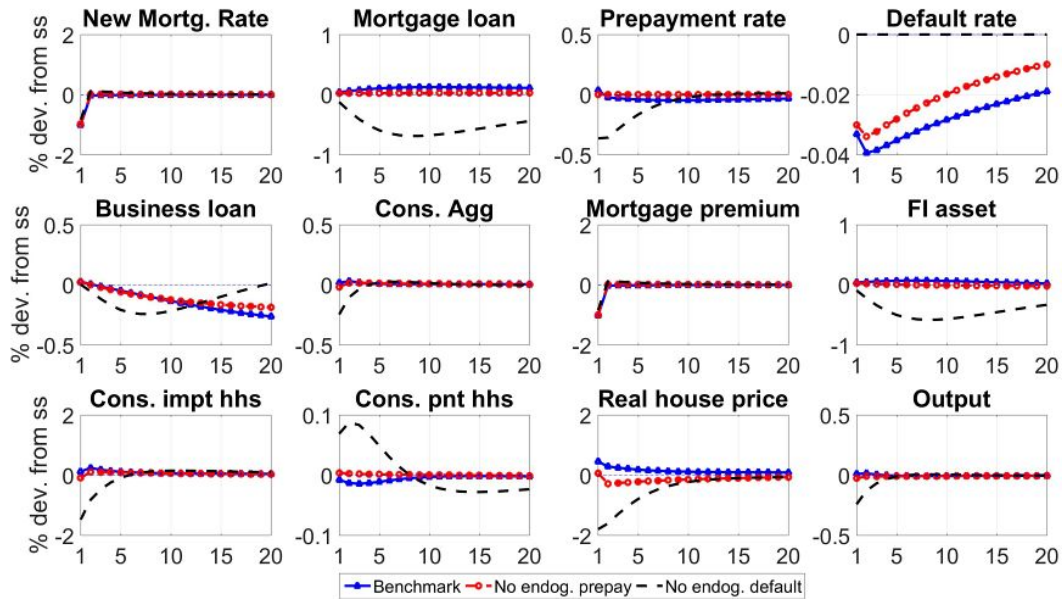
Given the qualitative similarities between these two cases, the effect of this unconventional monetary policy crucially depends on the mortgage credit channel. In the benchmark economy, the lower interest rate on the mortgage drives up the prepayment rate on impact as a massive wave of households take on new debt by taking advantage of this cheaper financing opportunity created by the policy intervention. However, this prepayment activity increase is short-lived as the rate is low for only one period. Even so, the benchmark economy has an apparent quantitative difference for most variables. For instance, the mortgage loan increase on impact in the benchmark economy is more than twice as large as the exogenous prepayment economy, and the difference widens to five to six times after about two years. The real house price and the financial intermediary’s asset responses exhibit qualitative differences: they rise on impact in the benchmark economy and are persistently positive, while in the alternative case their responses largely remain negative. Output also exhibits slightly different patterns between the two cases and is more prone to be positive in the benchmark case.

It is worth noting that the unconventional policy creates an opposite co-movement between the impatient and patient households’ consumption.<sup>65</sup> As the interest rate is cut lower by the policy intervention, the impatient households have cheaper access to mortgage credit, while the patient households, as the ultimate lender, receive less interest income. Since the impatient households have less income than the patient ones, the

<sup>65</sup> The opposite co-movement of consumption is also observed for the mortgage loan risk shock and other shocks creating redistribution between the two kinds of households in general, which is a property of this type of heterogeneous agent models. For the technology shock, the consumption comovement is positive. The conventional monetary policy can also drive the consumption for both households in the same direction: see [Pietruni and Signoretto \(2017\)](#) for a related discussion.

marginal propensity of consumption is also higher. Thus, the redistribution effect created by the policy can create an increase in aggregate consumption, depending on the relative population size between the two types of households. The endogenous prepayment mechanism intensifies the opposite co-movement in consumption, as households can optimally adjust the quantities of new loans and increase liabilities when rates are low.

Figure 15: Response to 1% Mortgage Loan Rate Decrease (cont.)

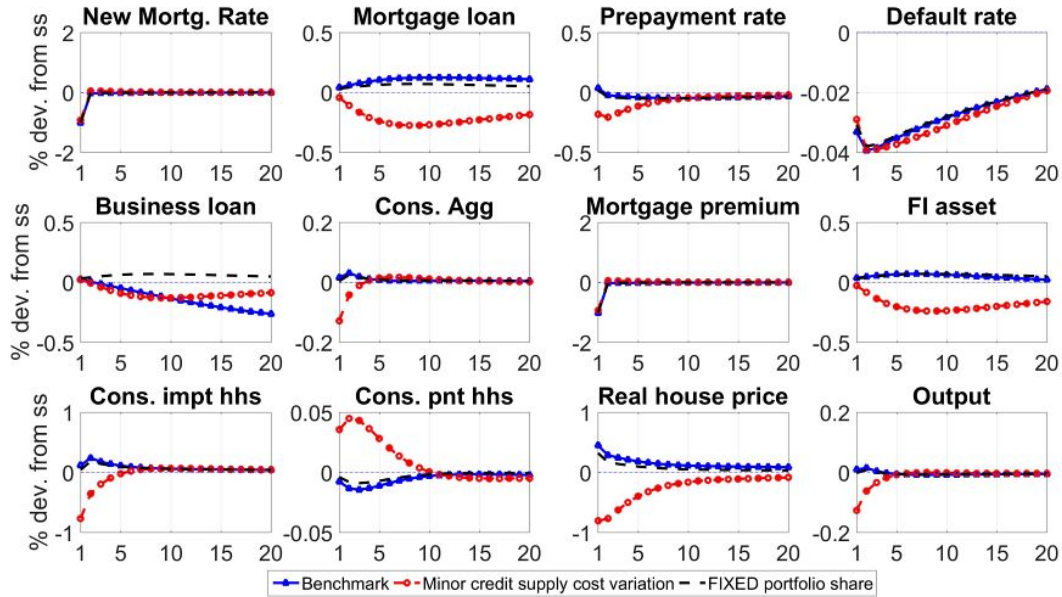


I also try to see the role played by the endogenous default mechanism. Figure 15 shows the result with an additional scenario shutting down endogenous default. This scenario is counterfactual, as the unconventional policy decreases the amount of mortgage credit in the economy and lowers the prepayment rate. Output and house prices also decrease significantly, and the consumption movements of the two types of households are opposite compared to the former two cases. In general, the unconventional policy fails to generate an expansionary outcome as observed (documented by Ferrante, 2018, for instance), mainly because the mortgage loan default can happen in the model, but the default choice is exogenous to the agents.

Lastly, figure 16 shows the role played by the credit supply mechanism in propagating this policy stimulus, including responses of the benchmark economy, the economy with minor credit supply cost variations,<sup>66</sup> and the economy with fixed asset portfolio composition in the financial intermediary. The second economy displays a counterfactual pattern, with responses qualitatively different to the benchmark case for several key variables such as the mortgage loan, consumption, real house price, and output. The reason is that the credit supply costs are not sensitive to the policy stimulus, and thus do not benefit much from the default rate decrease in the real sector. High intermediation costs hinder

<sup>66</sup>I calibrate the elasticities of supply costs to 0.5 for all three parameters, which are lower than their estimated posterior means.

Figure 16: Response to 1% Mortgage Loan Rate Decrease: Credit Supply Channels



the credit supply increase to the real sector and make the monetary policy ineffective. For the last case, the fixed portfolio share in the financial intermediary prohibits substitution between sectoral loans, and the stimulus effect of the policy is marginally dampened.

## 6 Conclusion

This paper builds a dynamic stochastic general equilibrium model and studies credit supply mechanisms subject to endogenous default and prepayment. The impatient households borrow long-term, prepayable and defaultable mortgage loans. Entrepreneurs also take defaultable business loans. These loan contracts are issued by the financial intermediary who supplies the credit subject to costs varying with the default and prepayment activities. The model also features loan contracts that influence the lender's net worth: unanticipated increases in default activities create losses to the financial intermediary. Last but not least, the financial intermediary endogenously chooses its asset allocation between the household and business sectors given the credit supply costs, subject to an adjustment friction.

To quantify these mechanisms, I estimate the model using Bayesian methods on U.S. data. The contractual losses in the model are consistent with most of the losses to the financial sector in the U.S. observed in the data, and also generate similar magnitudes of net worth fluctuations in the financial sector. The credit supply cost elasticities are estimated to be significantly positive, showing the presence of the credit supply cost variations empirically. The adjustment cost friction of portfolio adjustment is estimated to be moderate, indicating an active change of asset composition in the financial intermediation sector.



To this point, the paper takes a first step trying to understand the private asset portfolio adjustment of FI in a general equilibrium framework and its macroeconomic implications.

The framework is then employed to study the implications of the mortgage credit channel and credit supply mechanisms for conventional and unconventional monetary policies. For the outcome of conventional monetary policy, the source of the shock matters: under a TFP shock, the endogenous prepayment mechanism enhances the policy effectiveness, confirming the result in the study by [Greenwald \(2018\)](#) which introduces this mortgage credit channel into a general equilibrium study. However, an opposite effect will happen due to the same channel when a mortgage risk shock hits, as households reduce their financing needs under this shock. The endogenous default activity of households dampens these differences brought by the endogenous prepayment in general but does not overturn the qualitative results. Also, the credit supply costs varying with real activities and portfolio optimization of the financial intermediary both weaken the efficiency of the policy, indicating a more intense interest rate cut is needed in the presence of the credit supply mechanisms introduced by this paper. These observations may serve as reasons for the deep interest rate cuts seen in the last crisis.

For the unconventional monetary policy, the effectiveness crucially depends on the mortgage prepayment channel, as households can react to the incentive and strengthen the policy outcome. The unconventional policy also can result in a redistribution of wealth between households, as discussed in [Pietrunti and Signoretti \(2017\)](#), and this effect is enlarged by the endogenous prepayment activities. Also, the endogenous default setup is necessary according to this experiment, as a model with only exogenous default can generate a counterfactual result because of the non-optimized household behavior. Credit supply channels introduced by this paper make the unconventional policy more effective, showing the important role played by the financial intermediary in propagating the policy innovation.

Although this paper develops a rich framework, it still abstracts from channels important in the last crisis. For instance, the financial intermediary here does not have any agency problem on its liability side, which is one of the main issues encountered in the latest financial crisis. Studies including [Gertler and Karadi \(2011\)](#) and [Ferrante \(2018\)](#) investigate this question by endogenizing the agency problem in the banking sector. However, this issue and real sector default are somewhat isolated and treated as different fundamental sources leading to business cycle fluctuations. [Ricci and Tirelli \(2017\)](#) link these two sources in a reduced-form fashion by directly assuming correlations between the mortgage loan default and banking sector shocks. In future work, it would be interesting and important to examine the endogenous relationship between the real sector loss and the banking sector crisis. This study works towards this direction by making the financial intermediary incur losses when defaults happen more than expected.

Also, the model proposed by this paper is subject to the critique by [Shi \(2015\)](#), as for all fundamental shocks considered in the paper, the real house price and household loan-to-value ratio (representing the liquidity) move in opposite directions, violating the liquidity shock hypothesis (LSH). Given the difficulty in the ability of models with exogenously determined loan-to-value ratios to reconcile the data, the literature has sought alterna-

tive explanations. [Justiniano et al. \(2015\)](#) argue that an exogenous loosening of the credit standard can be one reason of the bust, reversing this mechanism to explain the empirical observation in the last recession. Works such as [Kurlat \(2013\)](#) and [Bigio \(2015\)](#) endogenize the loan-to-value ratio under imperfect information. In this study, I notice that the business sector can generate positive co-movement of the capital price and leverage given the business loan default shock. Therefore, this risk shock described in [Christiano et al. \(2014\)](#) with the BGG type contract could offer one explanation to the LSH. A similar type of contract for households may also solve this puzzle. Notice that the risk should matter for higher order approximations of the model solution, and agents display precautionary behavior when variances of exogenous disturbances increase, resulting in a simultaneous drop in asset prices and the loan-to-value ratio. Thus, a higher-order study of these types of models may offer a more satisfactory explanation of the LSH. These considerations are left for further research.

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